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MATHEMATICAL CHARACTERIZATION OF MECHANICAL BEHAVIOR
OF POROUS FRICTIONAL GRANULAR MEDIA

By

T. J. Chung and J. K. Lee

Final Technical Report

This research work was supported by the
National Aeronautics and Space Administration
under Contract NAS8-25102

Department of Engineering Mechanics
The University of Alabama in Huntsville
Huntsville, Alabama

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PREFACE

This report consists of two parts. Part I is concerned with the static displacement and stress fields. Part II describes the dynamic wheel-soil interaction. These studies were conducted during the period January 1, 1972 through October 31, 1972, under NASA Research Contract NAS8-25102 "Mathematical Characterization of Mechanical Behavior of Porous Frictional Granular Media," technically monitored by Dr. N. C. Costes, The Geotechnical Laboratory of the Marshall Space Flight Center, NASA, Huntsville, Alabama.

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ABSTRACT

A new definition of loading and unloading along the yield surface of Roscoe and Burland is introduced. This is achieved by noting that the strain-hardening parameter in the plastic potential function is deduced from the yield locus equation of Roscoe and Burland. The analytical results are compared with the experimental results for plate-bearing and cone-penetrometer problems and close agreements are demonstrated.

The second part of the reports deals with the wheel-soil interaction under dynamic loading. The rate-dependent plasticity or viscoelastoplastic behavior is considered. This is accomplished by the internal (hidden) variables associated with time-dependent viscous properties directly superimposed with inelastic behavior governed by the yield criteria of Roscoe and Burland. Effects of inertia and energy dissipation are properly accounted for. Exhaustive example problems are presented.

PART I

STATIC DEFORMATION AND STRESS FIELDS

I-1. INTRODUCTION

Recent achievements in the critical state soil mechanics advanced by Roscoe and others [1,2] have stimulated many other investigators searching for practical applications. Initial attempts have been made by Smith and Kay [3], Zienkiewicz [4], Chung and Lee [5], and Chung, Costes, and Lee [6] in the context of finite element techniques. The present study is an extension of [5,6] with some significant modifications in reference to interpretation of the yield criteria of Roscoe and Burland [1].

In the previous works [5,6], the authors considered the strain-hardening parameter to be controlled by the constant yield stress, an independent material parameter, in addition to the basic material properties M , λ and μ proposed by Roscoe and Burland [1]. However, in view of the fact that the equation of yield surface and subsequently the equation of yield locus as defined in [1] are based on the normality requirements of the plastic strain vector with strain-hardening phenomena incorporated in the plastic potential function, additional imposition of strain-hardening through a constant yield stress is unnecessary. Because the terms included in the plastic potential function [5,6] consists of deviatoric stress

invariant and the basic soil mechanics material properties (M, λ, κ) associated with the mean pressure the later contributions in the plastic potential function must provide strain-hardening behavior in the sense of classical incremental theory of plasticity. This argument leads to the standard manner of handling the plastic potential function in that the variation of the plastic potential function simply depends on the second deviatoric stress invariant and the strain-hardening parameter. If such variation is equal to zero we have a neutral loading, and the positive and negative values would indicate loading and unloading, respectively. The positive change of this potential function, therefore, shifts the yield locus in the deviatoric-mean stress space whose projection back to the void ratio - mean stress space lies entirely on the yield surface at all times.

The constitutive relationships and the finite element equations are derived as demonstrated earlier [5,6]. The plastic tangent stiffness matrix is updated for small increments of loading. The repetitive solution of the equilibrium equations continues until the total load is reached. Numerical examples for the plate-bearing and cone-penetrometer are presented to evaluate correctness of the procedure. Comparisons with test results indicate close agreements.

I-2. YIELD CRITERIA AND PLASTIC STIFFNESS

We record here the following basic assumptions of the critical state theory: (1) the soil material is continuously distributed over its whole volume with its behavior described by a macroscopic model; (2) the mechanical behavior of cohesive and cohesion-

less soil depends only on effective stresses independent of the presence or absence of pore pressures. The consequences of these assumptions lead to a complete description of soil behavior in a space of void ratio e , mean pressure p , and deviatoric pressure q . The deviatoric and volumetric strains corresponding to q and p along the yield locus are then related by means of the normality principle of plasticity theory as shown in Figure 1.

The mathematical model of pre-yield behavior may be based on the simple assumption of complete rigidity or elasticity, although some evidence exists of irrecoverable plastic shear distortion in this range [1]. For simplicity we may use the elasticity theory for the range of elastic wall (Figure 1).

To deal with irrecoverable volumetric and deviatoric strains and recoverable volumetric strains we turn to the equation of yield locus,

$$\frac{p}{p_0} - \frac{M^2}{M^2 + \eta^2} = 0 \quad (1)$$

where $\eta = q/p$; p_0 is the mean pressure corresponding to $q = 0$; and M is the slope η at the critical state line,

$$M = \frac{6 \sin \varphi}{3 - \sin \varphi} \quad (2)$$

in which φ is the angle of internal friction.

The incremental plastic (irrecoverable) volumetric strain is

$$dv^{(P)} = - \frac{de^{(P)}}{1 + e} \quad (3)$$

The overall void ratio change along the isotropic compression curve is

$$de = -\lambda \frac{dp_0}{p_0} \quad (4)$$

whereas the incremental recoverable void ratio is given by

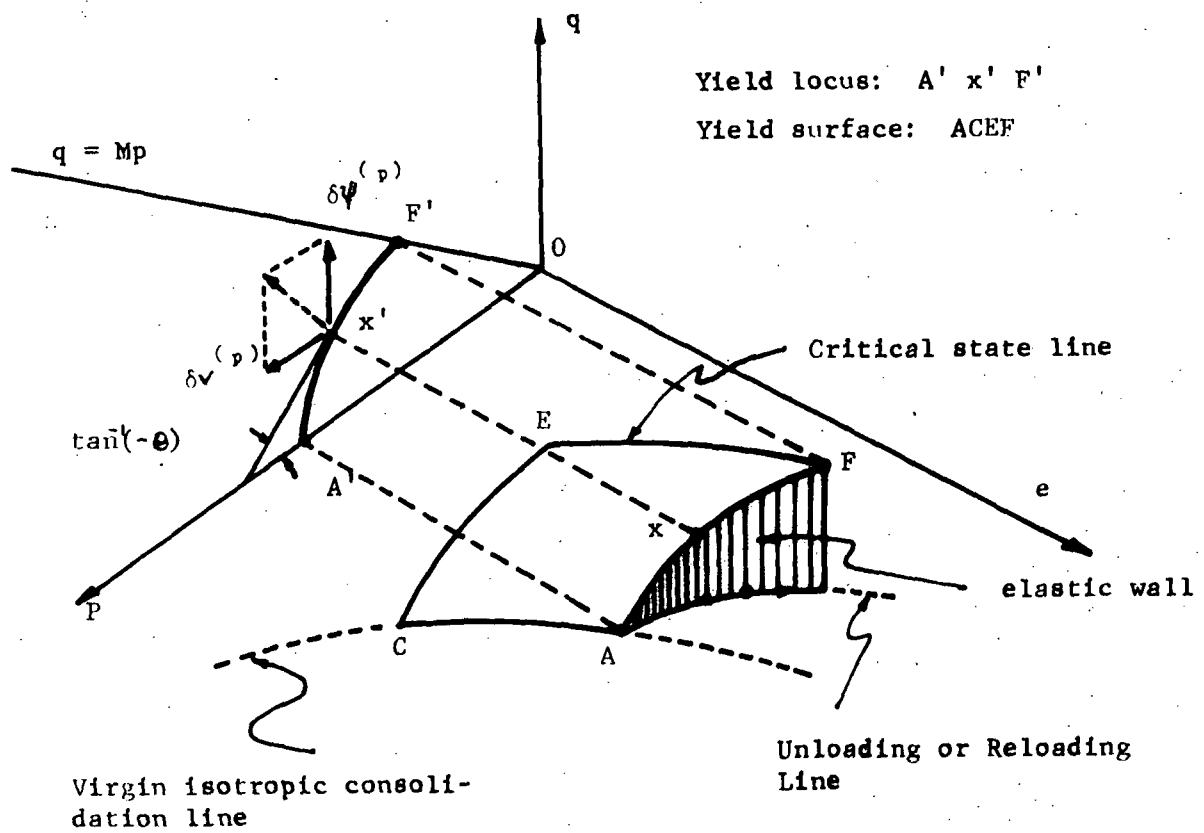


Figure 1: Yield Surface and Yield Locus (after Roscoe and Burland)

$$de^{(r)} = -\kappa \frac{dp_0}{p_0} \quad (5)$$

Here λ and κ are the compression index and swelling index, respectively.

The incremental irrecoverable void ratio is then obtained from (4) and

(5) as

$$de^{(p)} = -(\lambda - \kappa) \frac{dp_0}{p_0} \quad (6)$$

At this point we introduce the equation of yield surface in the form

[1]

$$\frac{p}{p_0} = \left(\frac{M^2}{M^2 + \eta^2} \right)^{\left(1 - \frac{\kappa}{\lambda}\right)} \quad (7)$$

in which p_0 is the equivalent pressure corresponding to that void on the virgin isotropic consolidation line whose projection to the $p - q$ space is P_0 .

Therefore, setting $p_0 = P_0$ in (7) leads to

$$P_0 = p \left(1 + \frac{\eta^2}{M^2} \right)^{(1 - \kappa/\lambda)} \quad (8)$$

Under triaxial compression, the second deviatoric stress invariant becomes

$$J = 1/3 (\sigma_{11} - \sigma_{33})^2 = 1/3 q^2 \quad (9)$$

which gives

$$q = \sqrt{3J} \quad (10)$$

Substituting (10) into (1) and rearranging yields

$$3J + p M^2 (p - P_0) = 0$$

or

$$3J - A^2 = 0 \quad (11)$$

where

$$A^2 = p M^2 (p - P_0) \quad (12)$$

It should be noted that (11) assumes the identical form as the plastic potential function $F(J, A)$ in the sense of classical incremental theory of plasticity,

$$F(J, A) = 3J - A^2 = 0 \quad (13)$$

The associated flow rule for the incremental plastic volumetric strain $dv^{(p)}$ and the incremental plastic deviatoric strain tensor $d\psi_{mn}^{(p)}$ may be written, respectively,

$$dv^{(p)} = \frac{\partial F}{\partial A} \frac{\partial A}{\partial p} d\lambda \quad (17)$$

$$d\psi_{mn}^{(p)} = \frac{\partial F}{\partial J} \frac{\partial J}{\partial \sigma_{mn}} d\lambda \quad (18)$$

in which $d\lambda$ is the positive constant. Here $dv^{(p)}$ may also be expressed in an alternate form from (3) and (6),

$$dv^{(p)} = \frac{\lambda - n}{1 + e} \frac{dp_0}{p_0} \quad (19)$$

Equating (17) and (19) and using (16) give

$$dv^{(p)} = \frac{(\lambda - \kappa) dp_0}{(1+e)p_0 \frac{\partial F}{\partial A} \frac{\partial A}{\partial p}} = \frac{-(\lambda - \kappa) \left(\frac{\partial F}{\partial J} dJ + \frac{\partial F}{\partial A} \frac{\partial A}{\partial p} dp \right)}{(1+e)p_0 \frac{\partial F}{\partial A} \frac{\partial A}{\partial p} \frac{\partial F}{\partial A} \frac{\partial A}{\partial p_0}} \quad (20)$$

The incremental total plastic strain tensor is given by

$$d\gamma_{mn}^{(p)} = d\psi_{mn}^{(p)} + 1/3 dv_{mn}^{(p)} \delta_{mn} \quad (21)$$

in which δ_{mn} is the Kronecker delta. Using (17) through (20) in (21) yields

$$d\gamma_{mn}^{(p)} = B_{mn} R_{\alpha\beta} d\sigma^{\alpha\beta} \quad (22)$$

where

$$B_{mn} = - \frac{\left(3 \frac{\partial J}{\partial \sigma_{mn}} + \frac{1}{3} \frac{\partial F}{\partial A} \frac{\partial A}{\partial p} \delta_{mn} \right) (\lambda - \kappa)}{p_0 (1+e) \frac{\partial F}{\partial A} \frac{\partial A}{\partial p} \frac{\partial F}{\partial A} \frac{\partial A}{\partial p_0}} \quad (23)$$

$$R_{\alpha\beta} d\sigma^{\alpha\beta} = \frac{\partial F}{\partial J} dJ + \frac{\partial F}{\partial A} \frac{\partial A}{\partial p} dp \quad (24)$$

also,

$$\frac{\partial F}{\partial J} dJ = 3 \frac{\partial J}{\partial \sigma_{mn}} d\sigma^{mn} = S_{mn} d\sigma^{mn} \quad (25a)$$

$$\frac{\partial F}{\partial A} \frac{\partial A}{\partial p} dp = (2pM^2 - p_0 M^2) dp \quad (25b)$$

$$\frac{\partial F}{\partial A} \frac{\partial A}{\partial p_0} = -M^2 p \quad (25c)$$

Substituting (25) into (23) gives

$$B_{mn} = \frac{S_{mn} + a \delta_{mn}}{b} \quad (26)$$

in which

$$a = \frac{M^2}{3} (2p - p_0) \quad (27)$$

$$b = 3a(1+e)M^2 p^2 p_0 / (\lambda - \kappa) \quad (28)$$

Similarly, $R_{\alpha\beta} d\sigma^{\alpha\beta}$ in (24) is given by

$$R_{\alpha\beta} d\sigma^{\alpha\beta} = (S_{\alpha\beta} + a\delta_{\alpha\beta}) d\sigma^{\alpha\beta} \quad (29)$$

where

$$S_{11} = 2\sigma_{11} - \sigma_{22} - \sigma_{33}$$

$$S_{22} = 2\sigma_{22} - \sigma_{11} - \sigma_{33}$$

$$S_{33} = 2\sigma_{33} - \sigma_{11} - \sigma_{22}$$

$$S_{12} = 6\sigma_{12}, S_{23} = 6\sigma_{23}, S_{31} = 6\sigma_{31}$$

The incremental total strain tensor dY_{mn} is the sum of the incremental elastic strain tensor $dY_{mn}^{(e)}$ and the incremental plastic strain tensor $dY_{mn}^{(p)}$.

Therefore,

$$dY_{mn}^{(e)} = dY_{mn} - dY_{mn}^{(p)} \quad (30)$$

The incremental total stress tensor $dY^{\alpha\beta}$ is then given by

$$d\sigma^{\alpha\beta} = D_{(e)}^{\alpha\beta mn} dY_{mn}^{(e)} \quad (31)$$

in which $D_{(e)}^{\alpha\beta mn}$ is the standard elasticity matrix. Substituting (30) and (22) into (31) yields

$$d\sigma^{\alpha\beta} = D^{\alpha\beta mn} (dY_{mn} - B_{mn} R_{1j} d\sigma^{1j}) \quad (32)$$

In view of (14b) and (24), and (32), we obtain

$$R_{rs} \left[D_{(e)}^{rs mn} (dY_{mn} - B_{mn} R_{\alpha\beta} d\sigma^{\alpha\beta}) \right] + \frac{\partial F}{\partial A} \frac{\partial A}{\partial p_0} dp_0 = 0$$

or

$$R_{rs} \left[D_{(e)}^{rs mn} (dY_{mn} - B_{mn} R_{\alpha\beta} d\sigma^{\alpha\beta}) \right] - R_{\alpha\beta} d\sigma^{\alpha\beta} = 0$$

from which

$$R_{\alpha\beta} d\sigma^{\alpha\beta} = \frac{R_{rs} D^{rs mn} dY_{mn}}{1 + R_{rs} B_{mn} D^{rs mn}} \quad (33)$$

Substituting (33) into (32) gives

$$d\sigma^{\alpha\beta} = \left(D_{(e)}^{\alpha\beta mn} + D_{(p)}^{\alpha\beta mn} \right) dY_{mn} \quad (34)$$

where

$$D_{(p)}^{\alpha\beta mn} = - \frac{D^{\alpha\beta kl} B_{kl} R_{1j} D^{1j mn}}{1 + B_{rs} R_{\alpha\beta} D^{rs mn}} \quad (35)$$

which is identical to the form obtained by the authors earlier [5,6].

Now, the yield criterion equation (14) is written as

$$dF = R_{\alpha\beta} d\sigma^{\alpha\beta} - M^2 p dp_0 \quad (36)$$

where dp_0 can be determined from (8),

$$dp_0 = g dp + h S_{1j} d\sigma_{1j} \quad (37)$$

in which

$$g = \left(1 + \frac{3J}{M^2 p^2}\right)^{(1-\kappa/\lambda)} - \frac{6J}{M^2 p^2} \left(1 - \frac{\kappa}{\lambda}\right) \left(1 + \frac{3J}{M^2 p^2}\right)^{(-\kappa/\lambda)} \quad (38a)$$

$$h = \frac{1}{M^2 p^2} \left(1 - \frac{\kappa}{\lambda}\right) \left(1 + \frac{3J}{M^2 p^2}\right)^{(-\kappa/\lambda)} \quad (38b)$$

Substituting these in (36) yields

$$dF = \left[S_{\alpha\beta} + a \delta_{\alpha\beta} - M^2 p \left(\frac{1}{3} g \delta_{\alpha\beta} + h S_{\alpha\beta} \right) \right] d\sigma^{\alpha\beta} \quad (39)$$

which is then used for determining the status of loading, neutral loading, and unloading as defined in (15 a, b, c).

I-3. APPLICATIONS

I.3.1 Plate Bearing

Based on the definition of yielding given by (15) the finite element computer program was written to solve boundary value problems. The program listing and data input format are given in Appendix 1. and Appendix 2, respectively.

Figure 2 shows the geometry of a plate bearing problem. The load-displacement curves for center of plate are shown in Figure 3 comparing the experimental results of Namiq [8]. It should be noted that the plane strain conditions of Namiq's experiments with a square box are approximated here in the analysis by an equivalent axis-symmetric cylindrical box. The material constants given by Namiq are angle of internal friction $\phi = 35^\circ$, initial void ratio $e = 0.875$, initial density $\gamma = 0.0147 \text{ N/cc}$. Other constants needed in this analysis are listed in Figure 3. It is seen that the load-displacement curve for the compression index $\lambda = 0.05$ follows very closely the experimental results whereas $\lambda = 0.13$ gives slightly larger displacements. It is interesting to note that from the void ratio-pressure curves given by Namiq the compression index can be estimated indeed to be approximately 0.05. Here the swelling index $\kappa = 0.003$ is used for both cases. For elastic behavior the soil modulus $E_s = 10 \text{ N/cm}^2$ and Poisson's ratio $\nu_s = .45$ are used.

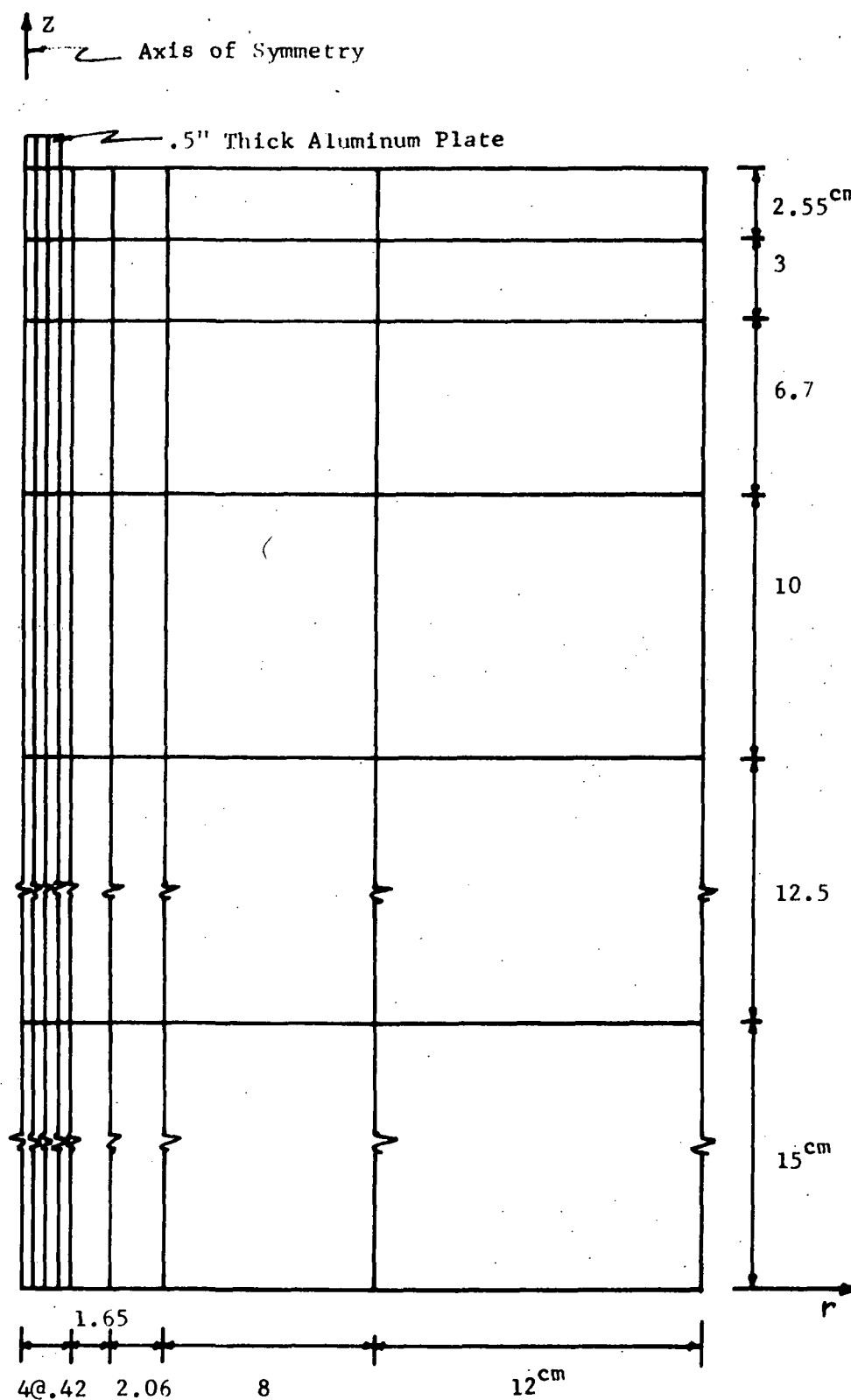


Figure 2: Plate Bearing Geometry

Legend	ϕ	e	ν	μ	λ	E	ν_s
————	35°	.875	.0147 N/cc	NA	NA	NA	NA
-----	"	"	"	.003	.05	10^{10} N/cm ²	.45
- · - · -	"	"	"	"	.13	"	"

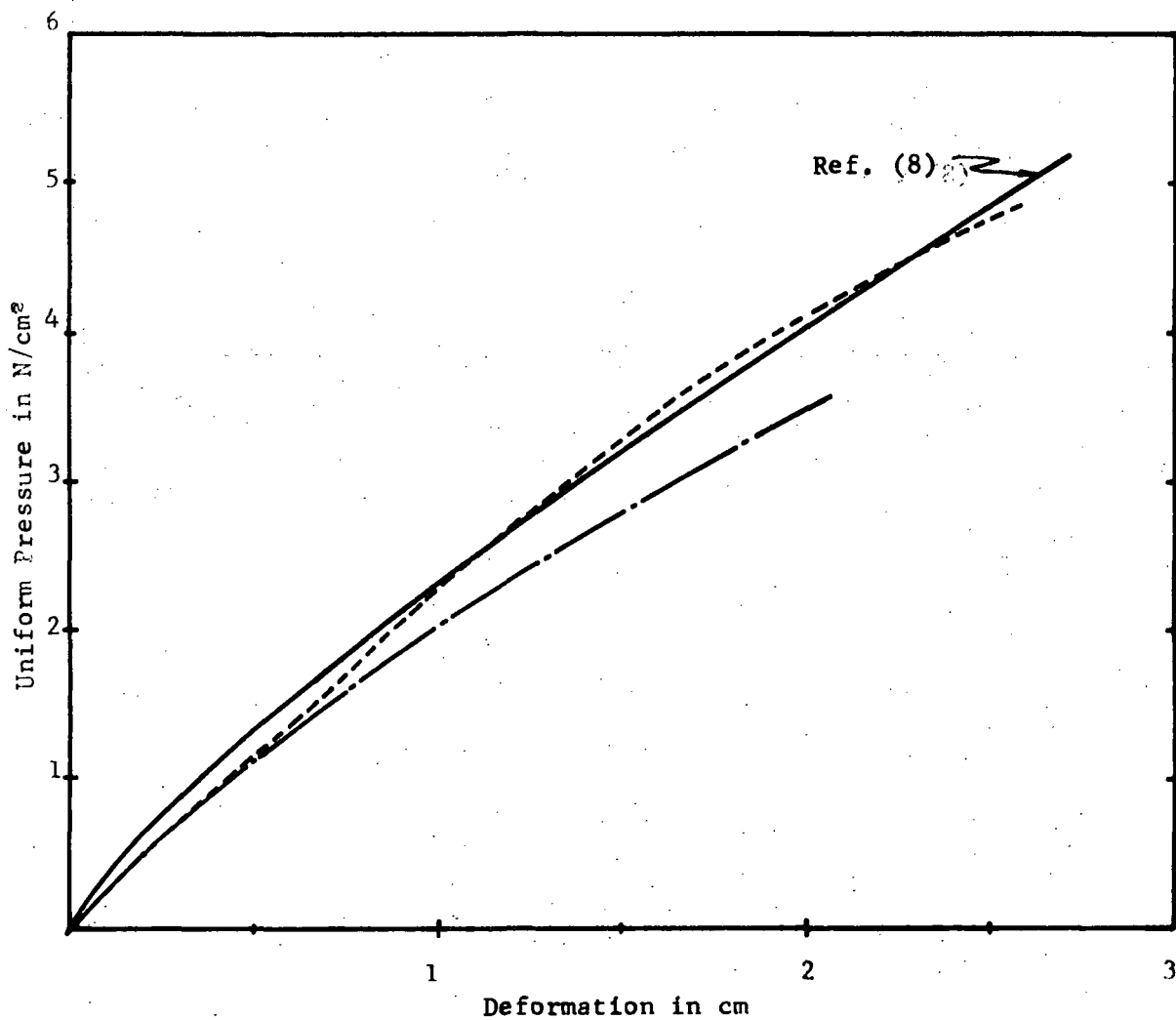


Figure 3: Deformation at Center of Plate

Deformed shapes of the finite elements adjacent to the bearing plate are shown in Figure 4 for the loading increments at $F = 2.5 \text{ N/cm}^2$ and $F = 5 \text{ N/cm}^2$. These results correspond to $\lambda = 0.05$ which gives the same displacement at the center of plate as Namiq. Unfortunately, however, no further comparison can be made as Namiq does not show such deformed shapes in his experimental results.

I.3.2 Cone-Penetrometer

The geometry for a cone-penetrometer problem is shown in Figure 5. Experiments for the cone-penetrometer were undertaken and the test set-up is shown in Figure 6. Both smooth and rough aluminum cones were used and loaded through the lunar soil simulants under the strain-controlled loading devices. These measurements are plotted in Figure 7 and compared with analytical results. The axisymmetric interface elements developed by Chung and Lee [5] are used to model contact areas between the cone and soil. Because of the lunar soil simulants being extremely soft compared with the metal cone the shear modulus and rotational modulus for the interface elements were set equal to zero. Experimentally determined material constants for the lunar soil simulants used in the tests are also given in Figure 7. The same material constants were used in the analysis with the exceptions of soil modulus $E_s = 10 \text{ N/cm}^2$ and Poisson's ratio $\nu_s = .45$. The analytical solution gives results somewhere between the rough and smooth cones.

The deformed geometry of soil is shown in Figure 8. For excessive alterations of finite elements in shape it would appear that

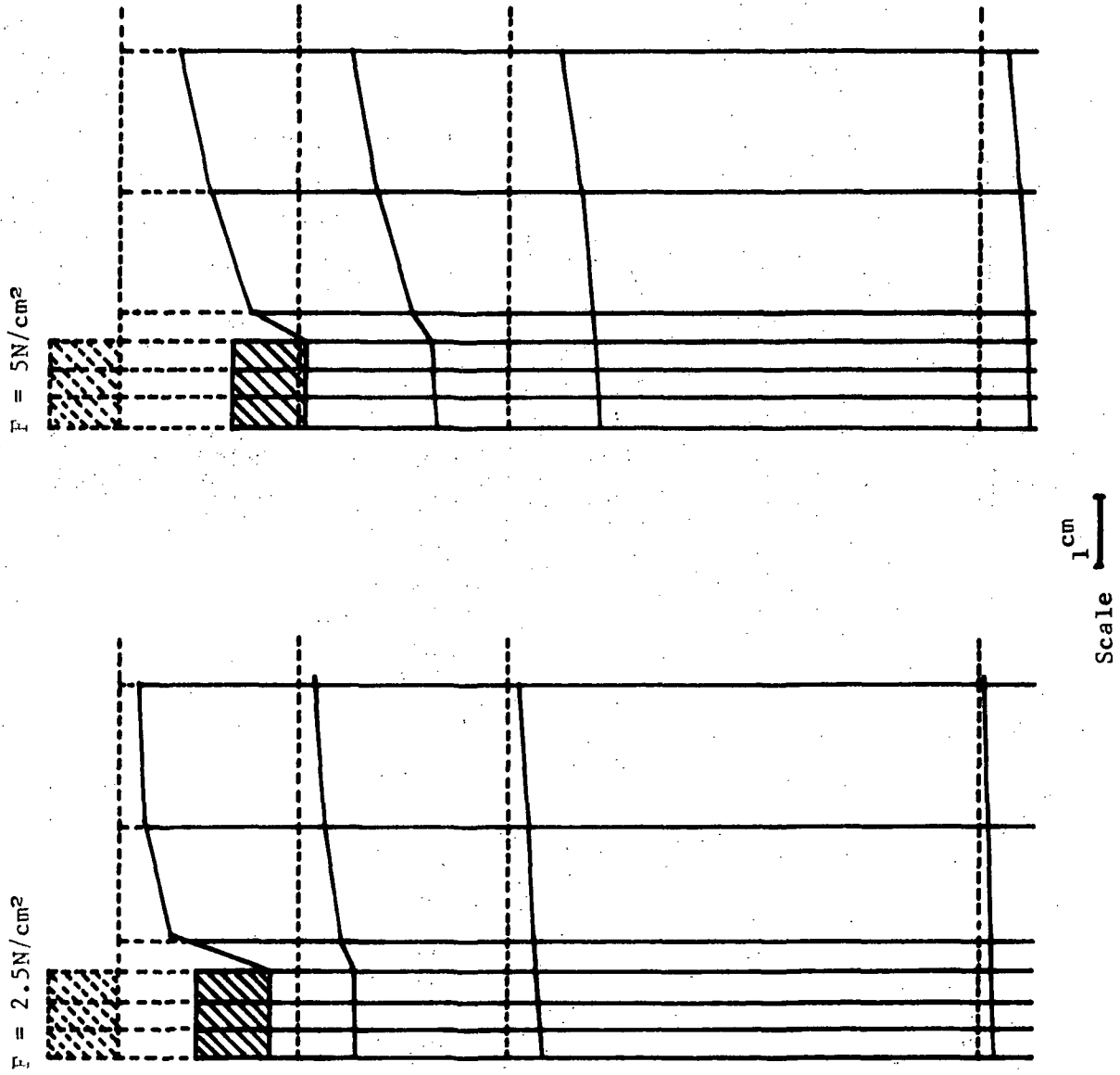


Figure 4: Deformed Geometry

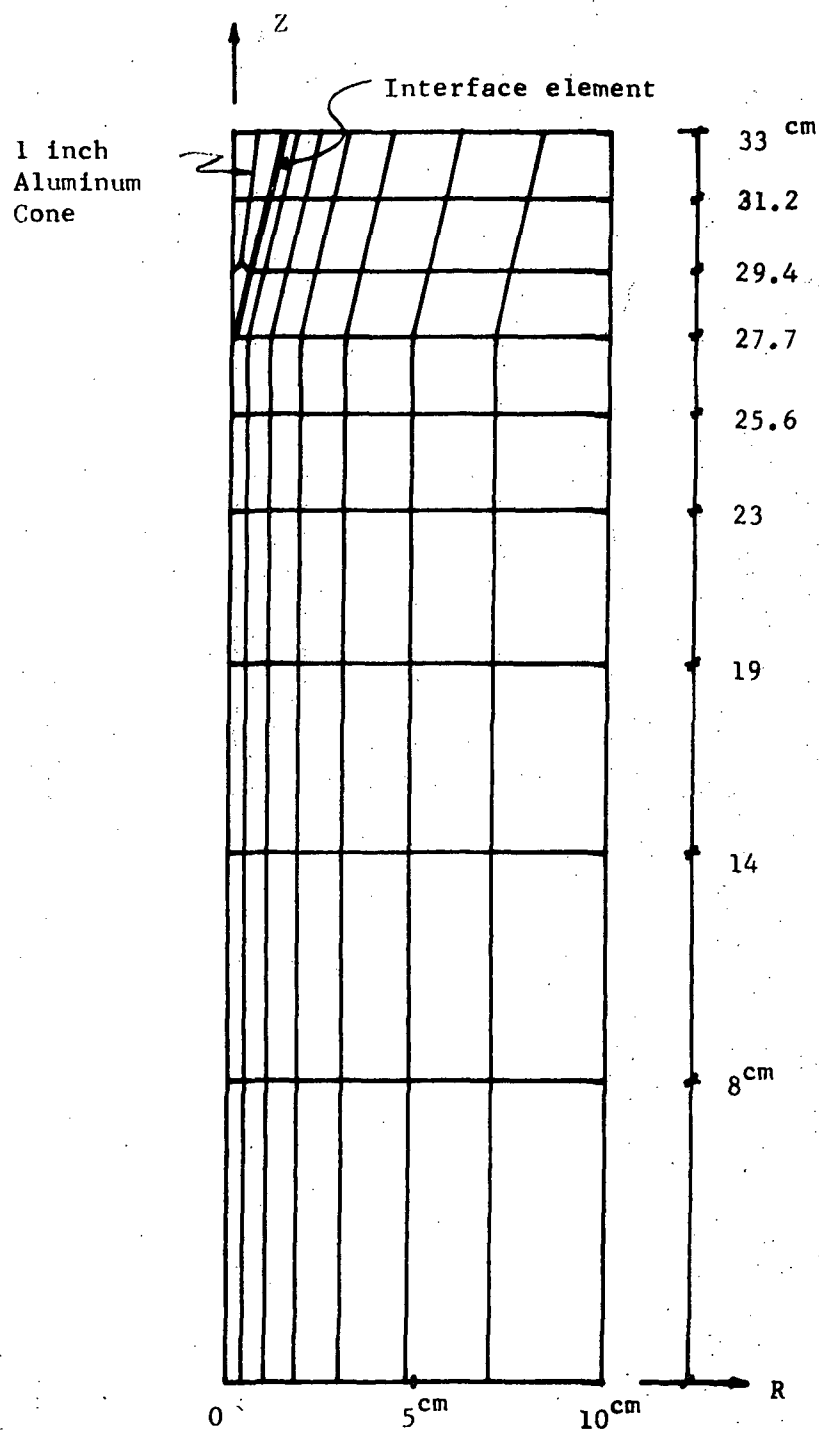
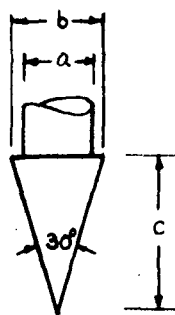
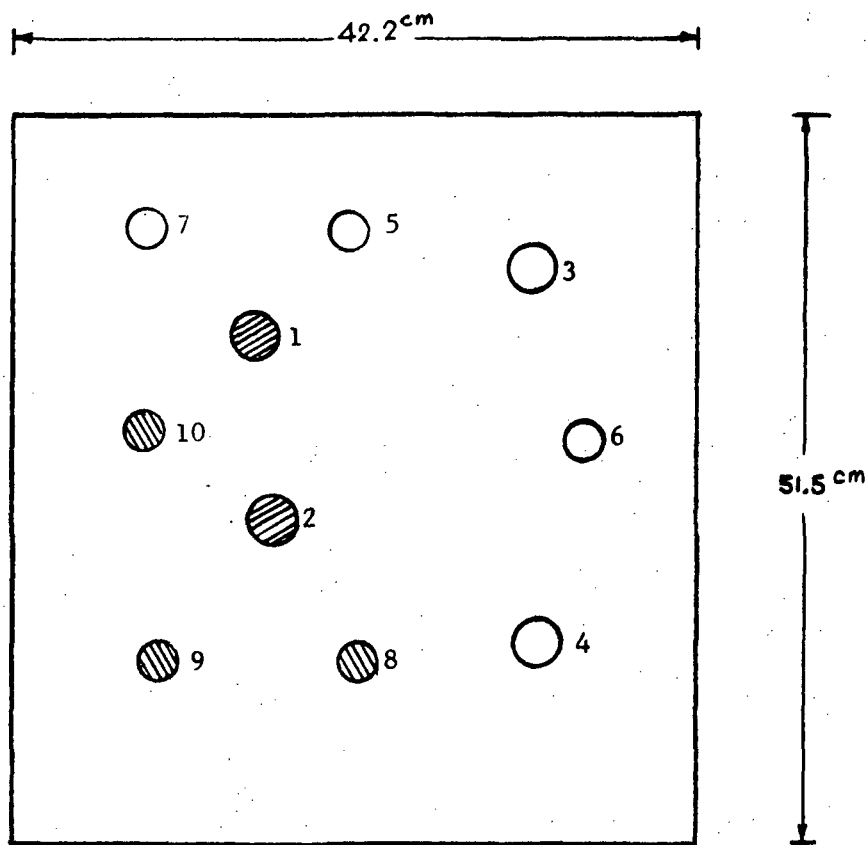




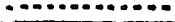



Figure 5: Cone-Penetrometer Geometry



- | | | |
|--------|--------------------------------|---|
| 1,2 | 1 inch Rough Cone | } $a=2.54^{\text{cm}}$, $b=2.82^{\text{cm}}$, $c=5.3^{\text{cm}}$ |
| 3,4 | 1 inch Smooth Cone | |
| 5,6,7 | $\frac{1}{2}$ inch Smooth Cone | } $a=1.7^{\text{cm}}$, $b=2^{\text{cm}}$, $c=3.9^{\text{cm}}$ |
| 8,9,10 | $\frac{1}{2}$ inch Rough Cone | |

Figure 6: Cone-Penetrometer Tests

Legend	Description
	Average of 1/2" Rough Cone Tests
	" " " " " "
	" " 1/2" Smooth Cone Tests
	" " " " " "
	Finite Element Solution (1), $\lambda = .07$ *
	" " (2), $\lambda = .13$ *

* Other constants used are:

$\alpha = .006$, $\phi = 35^\circ$, $e = .76$, $\nu = .0157$ N/cc

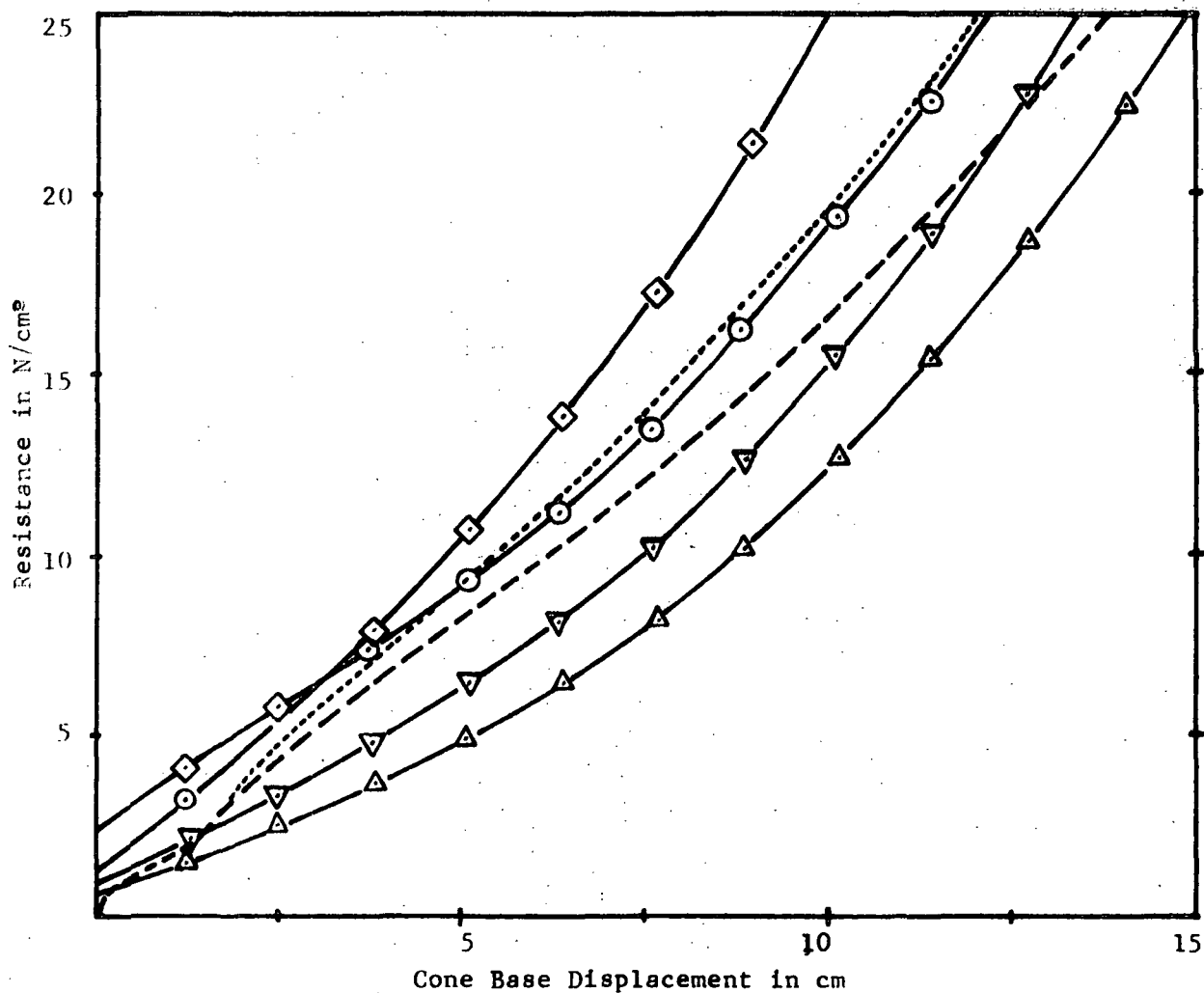


Figure 7: Force-Displacement Curves for Cone Penetrometer

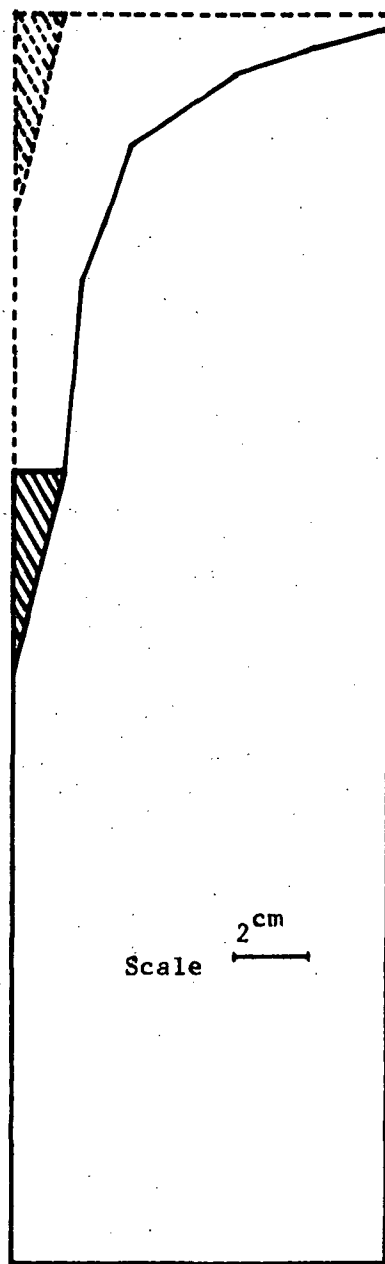


Figure 8: Deformed Configuration for Finite Element
Solution (1) at $F = 25 \text{ N/cm}^2$

renumbering of nodes is necessary to update the stiffness matrix based on new geometry. It is believed that such treatment would improve the solution considerably.

I-4. CONCLUSIONS

A new definition of loading and unloading along the yield surface of Roscoe and Burland is introduced. This is done by noting the strain-hardening parameter in the plastic potential function. With the differential of the plastic potential function with respect to the second deviatoric stress invariant and the strain-hardening parameter being positive or negative the manner of loading and unloading is clearly determined. This is an improvement from the previous definition of yielding through a constant yield stress.

The forms of plastic stiffness matrix and the finite element equations, however, are unchanged. Applications of the present analytical formulation to a number of boundary value problems are presented. The analytical results for the plate bearing and cone-penetrometer problems indicate good agreements with the experimental results.

Our ultimate goal is to characterize the material parameters of the lunar soil. Such a task depends on correct constitutive relationships and a computational scheme which provides the results of load-deformation. With this facility available exhaustive computer runs

for various combinations of material constants are to be compared with the data brought back from the lunar exploration. To this end the present study has provided the basic analytical tool to prepare for such an undertaking.

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APPENDIX 1

COMPUTER PROGRAM LISTING
(Static Analysis - Axisymmetric)

WELT,IDL DECK,,SYSTOP
ELT 005-11/24-14:39

```
000001 000 WELT,SIH NASA*TPF$.MAIN,,132656133010
000002 000 C-----00000100
000003 000 C 00000200
000004 000 C THE FINITE ELEMENT ANALISYS OF AXISYMMETRIC SOIL MEDIUM 00000300
000005 000 C BY A SOIL PLASTICITY THEORY 00000400
000006 000 C 00000500
000007 000 C-----00000600
000008 000 C 00000700
000009 000 PARAMETER NODS=300,NELS=260,NF=20000,MAX=600 00000800
000010 000 C 00000900
000011 000 COMMON /BLK0/ TITLE(20),INODE,NELEM,NAPC,NBC,NINCR,NCYCL,EPSON 00001000
000012 000 COMMON /BLK1/ W(6),H(6),AR(4),BR(4),CR(4),AZ(4),BZ(4),CZ(4), 00001100
000013 000 * BN(4),CN(4),DN(4),TYPEA(4,4),TYPEB(4,4),TYPEC(4,4),TYPEE(4,4), 00001200
000014 000 * TYPEF(4,4),YPEG(4,4),AO,BO,CO,RT,RB,RA,RC,IC,JC,KC,LC,NEL 00001300
000015 000 COMMON /BLK2/ ID(NODS,2),IJKL(NELS,4),DEI,DE2 00001400
000016 000 COMMON /BLK3/ XK(NF),APF(MAX),IMAX,IHR,IHRI,LT,LAST 00001500
000017 000 COMMON /BLK4/ STRS(NELS,4),DMAT(NELS,4,4),DELNM1(MAX),POP 00001600
000018 000 COMMON /BLK5/ DE(NELS,4,4),SIGBA(NELS),DSIGBA(NELS),DELL,YSTRS, 00001700
000019 000 1 FINC,FN,ULOAD,FEL,PMAX,ULMAX 00001800
000020 000 COMMON /BLK6/ SIGR,SIGZ,SIGT,TAUZR,D(4,4),STIFF(8,8),KK(NELS,8), 00001900
000021 000 1 R(NODS),Z(NODS),IUTDIS(MAX) 00002000
000022 000 COMMON /BLK7/ USTRS(NELS,4),ARM(NELS,4),AZM(NELS,4),RTT(NELS), 00002100
000023 000 1 AOJ(NELS) 00002200
000024 000 COMMON/BLK8/INCR,PDEPTH(NELS),VOIDI,ALAMDA,DEPTH,PP 00002300
000025 000 COMMON /BLK9/ PI,SMALLK,CK,BETA,PO,NFREE,NELST,ICASE,NRIGD 00002400
000026 000 COMMON /BLK10/ FRCK(20,8,8),TR(20,8,8),XXL(20),DZZ(20),DRR(20), 00002500
000027 000 1 POR(20),DELTS,EC,XNUC,DELD 00002600
000028 000 COMMON /BLK11/ VOID(NELS),DGAM(NELS,4) 00002700
000029 000 COMMON /BLK12/ SIGMX(NELS),DEP(NELS),EP(NELS),DEGST2(NELS),ES 00002800
000030 000 COMMON /GMTRY/ RO(NODS),ZO(NODS) 00002900
000031 000 C-----00003000
000032 000 C 00003100
000033 000 NTAPE = 2 00003200
000034 000 C 00003300
000035 000 C-----00003400
000036 000 C 00003500
000037 000 CALL SETUP 00003600
000038 000 C 00003700
000039 000 C INITIALIZES NECESSARY CONSTANTS FOR INTEGRATION SCHEME. 00003800
000040 000 C-----00003900
000041 000 C 00004000
000042 000 CALL INPUT(NTAPE) 00004100
000043 000 ISHEAR = 1 00004200
000044 000 C 00004300
000045 000 C 00004400
000046 000 FN = FEL 00004500
000047 000 C 00004600
000048 000 C-----00004700
000049 000 C 00004800
000050 000 C START MAIN ITERATION LOOP. 00004900
000051 000 C 00005000
000052 000 C-----00005100
000053 000 DO 990 NI = 1,NINCR 00005200
000054 000 C 00005300
000055 000 FN = FN + FINC 00005400
```


000056	000	ITER = 0	00005500
000057	000		00005600
000058	000	C IF(NI.EQ.1) GO TO 950	00005700
000059	000	ISHEAR = NI	00005800
000060	000	CALL ZERO(XK,NF,1)	00005900
000061	000	REWIND NTAPE	00006000
000062	000	C	00006100
000063	000	DO 940 NEL = 1,NELEM	00006200
000064	000	IC = IJKL(NEL,1)	00006300
000065	000	JC = IJKL(NEL,2)	00006400
000066	000	KC = IJKL(NEL,3)	00006500
000067	000	LC = IJKL(NEL,4)	00006600
000068	000	NF1 = NEL - NRIGD	00006700
000069	000	IF(NEL.LE.NRIGD) GO TO 938	00006800
000070	000	SIGZ = STRS(NEL,1) + DSTRS(NEL,1) / 2.	00006900
000071	000	SIGX = STRS(NEL,2) + DSTRS(NEL,2) / 2.	00007000
000072	000	SIGY = STRS(NEL,3) + DSTRS(NEL,3) / 2.	00007100
000073	000	TAUZR = STRS(NEL,4) + DSTRS(NEL,4) / 2.	00007200
000074	000	IF(NBC.NE.U.AND.NF1.LE.NBC) GO TO 899	00007300
000075	000	C-----	00007400
000076	000	C	00007500
000077	000	CALL DMATRX(NI)	00007600
000078	000	C	00007700
000079	000	C CALCULATE STRESS DEPENDENT MATERIAL PROPERTY MATRIX (D).	00007800
000080	000	C-----	00007900
000081	000	C	00008000
000082	000	GO TO 837	00008100
000083	000	C	00008200
000084	000	938 DO 400 I = 1,4	00008300
000085	000	DO 400 J = 1,4	00008400
000086	000	DMAT(NEL,I,J) = DE(NEL,I,J)	00008500
000087	000	400 D(I,J) = DMAT(NEL,I,J)	00008600
000088	000	837 CALL STIFF2(NI,NTAPE)	00008700
000089	000	GO TO 838	00008800
000090	000	C	00008900
000091	000	899 CALL FRICTN(IC,KC,NEL,NF1,ISHEAR,VOIDI)	00009000
000092	000	C	00009100
000093	000	838 CALL ASSEMB(NEL,NF1)	00009200
000094	000	940 CONTINUE	00009300
000095	000	950 CONTINUE	00009400
000096	000	ITER = ITER + 1	00009500
000097	000	INCR = ITER	00009600
000098	000	C	00009700
000099	000	CALL DISPL(NFREE,NI,INODE,INCR)	00009800
000100	000	C	00009900
000101	000	DO 340 I = 1,INODE	00010000
000102	000	JJ = I * 2	00010100
000103	000	II = JJ - 1	00010200
000104	000	Z(I) = ZO(I) + TOTDIS(II) + XK(LAST+II)	00010300
000105	000	340 R(I) = RO(I) + TOTDIS(JJ) + XK(LAST+JJ)	00010400
000106	000	C	00010500
000107	000	CALL STRAIN(NI,ITER)	00010600
000108	000	C	00010700
000109	000	C SUMMING OF STRESSES AND DISPLACEMENTS FOR EACH INCREMENTAL STEP	00010800
000110	000	DO 310 IIT = 1,NFREE	00010900
000111	000	310 TOTDIS(IIT) = TOTDIS(IIT) + XK(IIT+LAST)	00011000
000112	000	DO 329 I = 1,NELEM	00011100

000113	000	DO 329 J = 1,4	00011200
000114	000	329 STRS(I,J) = STRS(1,J) + DSTRS(I,J)	00011300
000115	000	C	00011400
000116	000	WRITE(6,689) NINCR,N1,FN	00011500
000117	000	DO 320 I = 1,INODE	00011600
000118	000	II = (I-1) * 2 + 1	00011700
000119	000	JJ = II + 1	00011800
000120	000	320 WRITE(6,690) I,(TOTDIS(LX),LX = II,JJ)	00011900
000121	000	WRITE(6,691)	00012000
000122	000	DO 330 I = 1,NELEM	00012100
000123	000	330 WRITE(6,692) I,(STRS(I,J),J=1,4)	00012200
000124	000	IF(FN.GT.PMAX) STOP LOADMX	00012300
000125	000	990 CONTINUE	00012400
000126	000	C	00012500
000127	000	C	00012600
000128	000	500 FORMAT(10I5)	00012700
000129	000	600 FORMAT(//, ' ITER',15, ' DMAX',E12.5, ' DE2',E12.5)	00012800
000130	000	689 FORMAT(1H1,10X, ' TOTAL DISPLACEMENT NO. OF INCREMENTAL STEPS	00012900
000131	000	*,215//5X,'NODE',5X,'Z - DISPL',20X,'R - DISPL',5X,'FN =',E12.5/)	00013000
000132	000	690 FORMAT(4X,15,E15.7,10X,E15.7)	00013100
000133	000	691 FORMAT(///,10X, ' TOTAL STRESSES//5X,'ELEM',5X,'SIGMA - Z',T28,	00013200
000134	000	1'SIGMA - R',T42,'TANGENTIAL',T58,'TAU - ZR')	00013300
000135	000	692 FORMAT(18,4F14.6)	00013400
000136	000	693 FORMAT(//, ' TAU, SIG, RAT, DELTS, ISHEAR, ',4E12.5,15//)	00013500
000137	000	694 FORMAT(//20X,'NEW GEOMETRY AT THE END OF LOAD INCR.',15//)	00013600
000138	000	695 FORMAT(110,2F15.6)	00013700
000139	000	STOP	00013800
000140	000	END	00013900
000141	000	WELT,SIH NASA*TPFS,INPUT,,,132661133010	
000142	000	SUBROUTINE INPUT(NTAPE)	00000100
000143	000	C-----	00000200
000144	000	PARAMETER NODS=300,NELS=260,NF=20000,MAX=600	00000300
000145	000	C-----	00000400
000146	000	COMMON /BLK0/ TITLE(20),INODE,NELEM,NAPC,NBC,NINCR,NCYCL, EPSLON	00000500
000147	000	COMMON /BLK1/ W(6),H(6),AR(4),BR(4),CR(4),AZ(4),BZ(4),CZ(4),	00000600
000148	000	* BN(4),CN(4),DN(4),TYPEA(4,4),TYPEB(4,4),TYPEC(4,4),TYPEE(4,4),	00000700
000149	000	* TYPEF(4,4),TYPEG(4,4),AO,BO,CO,RT,RB,RA,RC,IC,JC,KC,LC,NEL	00000800
000150	000	COMMON /BLK2/ ID(NODS,2),IJKL(NELS,4),DE1,DE2	00000900
000151	000	COMMON /BLK3/ XK(NF),APF(MAX),IMAX,IHR,IHBI,LT, LAST	00001000
000152	000	COMMON /BLK4/ STRS(NELS,4),DMAT(NELS,4,4),DELNM1(MAX),POP	00001100
000153	000	COMMON /BLK5/ DE(NELS,4,4),SIGBA(NELS),DSIGBA(NELS),DELL,YSTRS,	00001200
000154	000	1 FINC,FN,ULOAD,FEL,PMAX,ULMAX	00001300
000155	000	COMMON /BLK6/ SIGR,SIGZ,SIGT,TAUZR,D(4,4),STIFF(8,8),KK(NELS,8),	00001400
000156	000	1 R(NODS),Z(NODS),TOTDIS(MAX)	00001500
000157	000	COMMON/BLK8/INCR,PDEPTH(NELS),VOIDI,ALAMDA,DEPTH,XMS	00001600
000158	000	COMMON /BLK9/ PI,SMALLK,CK,BETA,PO,NFREE,NELST,ICASE,NRIGD	00001700
000159	000	COMMON /BLK10/ FRCK(20,8,8),TR(20,8,8),XXL(20),DZZ(20),DRR(20),	00001800
000160	000	1 POR(20),DELTS,EC,XNUC,DELD	00001900
000161	000	COMMON /BLK11/ VOID(NELS),DGAM(NELS,4)	00002000
000162	000	COMMON /BLK12/ SIGMX(NELS),DEP(NELS),EP(NELS),DEQST2(NELS),ES	00002100
000163	000	COMMON /GMTRY/ RO(NODS),ZO(NODS)	00002200
000164	000	C-----	00002300
000165	000	REWIND NTAPE	00002400
000166	000	READ(5,510) (TITLE(I),I=1,20)	00002500
000167	000	READ(5,500) INODE,NELEM,NAPC,NBC,NINCR,NCYCL,ICASE,NRIGD,NULOAD	00002600
000168	000	READ(5,530) YSTRS,DELL,ZETA,PMAX,DI,MAX	00002700
000169	000	READ(5,511) DZI,DR1	00002800

000170	000	C	DZI = SHEAR MOD. FOR INTERFACE ELEMENTS.	00002900
000171	000	C	DRI = NORMAL FOR INTERFACE ELEMENTS.	00003000
000172	000		READ(5,511) EC,XNUC,ES,XNUS	00003100
000173	000		READ(5,530) PI,SMALLK,XI,VOIDI,PO,DEPTH,ALAMDA,EPSLON	00003200
000174	000		WRITE(6,530) PI,SMALLK,XI,VOIDI,PO,DEPTH,ALAMDA,EPSLON	00003300
000175	000		WRITE(6,630) YSTRS,DELL	00003400
000176	000		WRITE(6,631) ZETA,DZI,DRI	00003500
000177	000	631	FORMAT(//, ZETA, MOD. FOR INTERFACE ELEM., SHEAR, NORMAL,	00003600
000178	000	1	3F15.6)	00003700
000179	000		POP = EXP(1.-SMALLK/ALAMDA)	00003800
000180	000		II = 0	00003900
000181	000		DO 101 I = 1,INODE	00004000
000182	000		DO 101 J = 1,2	00004100
000183	000		II = II + 1	00004200
000184	000	101	ID(I,J) = II	00004300
000185	000		BETA=ALAMDA-SMALLK	00004400
000186	000		PI=PI*3.14159/180.	00004500
000187	000		EPSLON = EPSLON * 3.14159 / 180.	00004600
000188	000		DELTS = TAN(EPSLON)	00004700
000189	000		PI=SIN(PI)	00004800
000190	000	C	PI IS SINE(PI)	00004900
000191	000		XM = 6.*PI/(3.-PI)	00005000
000192	000		XMS = XM * XM	00005100
000193	000		WRITE(6,600) (TITLE(I),I=1,20)	00005200
000194	000		DO 100 I = 1,INODE	00005300
000195	000		READ(5,520) Z(I),R(I),IZ,IR	00005400
000196	000		ZO(I) = Z(I)	00005500
000197	000		RO(I) = R(I)	00005600
000198	000		IF(IZ.NE.0) ID(I,1) = 0	00005700
000199	000		IF(IR.NE.0) ID(I,2) = 0	00005800
000200	000	100	WRITE(6,620) I,Z(1),R(1),IZ,IR	00005900
000201	000		WRITE(6,501) INODE,NELEM,NAPC,NINCR,NCYCL	00006000
000202	000		NFREE = INODE * 2	00006100
000203	000	C		00006200
000204	000		WRITE(6,651)	00006300
000205	000		READ(5,540) ((IJKL(NEL,J),J=1,4),NEL=1,NELEM)	00006400
000206	000		WRITE(6,650) (NEL,(IJKL(NEL,J),J=1,4),NEL=1,NELEM)	00006500
000207	000	C		00006600
000208	000	C	FIND HALF BAND WIDTH AND ACTUAL SIZE OF MATRIX (XK)	00006700
000209	000	C		00006800
000210	000		IMAX = 0	00006900
000211	000		DO 800 NEL = 1,NELEM	00007000
000212	000		DO 700 I = 1,4	00007100
000213	000		IN = IJKL(NEL,I)	00007200
000214	000		KK(NEL,I) = ID(IN,1)	00007300
000215	000	700	KK(NEL,I+4) = ID(IN,2)	00007400
000216	000		DO 7999 I = 1,2	00007500
000217	000		II = I + 1	00007600
000218	000		DO 7999 J = II,4	00007700
000219	000		IDIF = IJKL(NEL,I) - IJKL(NEL,J)	00007800
000220	000		IF(IDIF.LT.0) IDIF = -IDIF	00007900
000221	000	7999	IF(IDIF.GT.IMAX) IMAX = IDIF	00008000
000222	000	800	CONTINUE	00008100
000223	000		IF(INBC.NE.0) READ(5,500) ((ID(I,J),J=1,2),I=1,NBC)	00008200
000224	000	C	IMAX = MAX DIFFERENCE IN ADJACENT NODE NO.	00008300
000225	000		IHB = (IMAX + 1) * 2	00008400
000226	000		IHBI = IHB - 1	00008500

000227	000	LT = IHB * IHB1 / 2	00008600
000228	000	LAST = LT + (NFREE - IHRI) * IHB	00008700
000229	000	WRITE(6,640) IMAX,IHB,LT,LAST	00008800
000230	000	C-----	00008900
000231	000	IF(NRIGD.NE.0) CALL ELASTC(D,NRIGD,EC,XNUC)	00009000
000232	000	C-----	00009100
000233	000	NLST = NELEM - NRIGD - NBC	00009200
000234	000	DO 900 NEL = 1,NELEM	00009300
000235	000	IC = IJKL(NEL,1)	00009400
000236	000	JC = IJKL(NEL,2)	00009500
000237	000	KC = IJKL(NEL,3)	00009600
000238	000	LC = IJKL(NEL,4)	00009700
000239	000	IF(NEL.LE.NRIGD) GO TO 898	00009800
000240	000	NFT = NEL - NRIGD	00009900
000241	000	IF(NFT.GT.NRC) GO TO 896	00010000
000242	000	DZL(NFT) = DZI	00010100
000243	000	DRK(NFT) = DRI	00010200
000244	000	CALL FRICTN(IC,KC,NEL,NFT,0,VOIDI)	00010300
000245	000	GO TO 899	00010400
000246	000	896 IF(ICASE. NE. 0) GO TO 897	00010500
000247	000	C	00010600
000248	000	CALL AREA(A,IC,JC,KC,LC,AREA)	00010700
000249	000	VOIDR = VOIDI	00010800
000250	000	VOID(NEL) = VOIDR	00010900
000251	000	DELNM1(NEL) = AREA	00011000
000252	000	C	00011100
000253	000	897 NELST = NRIGD + NBC + 1	00011200
000254	000	IF(NEL.EQ.NELST)CALL ELASTC(D,NLST,ES,XNUS)	00011300
000255	000	C	00011400
000256	000	898 DO 111 I = 1,4	00011500
000257	000	DO 111 J = 1,4	00011600
000258	000	DE(NEL,I,J) = D(I,J)	00011700
000259	000	111 DMAT(NEL,I,J) = D(I,J)	00011800
000260	000	C	00011900
000261	000	CALL STIFF1(NTAPE)	00012000
000262	000	CALL STIFF2(0,NTAPE)	00012100
000263	000	899 CALL ASSEMB(NEL,NFT)	00012200
000264	000	C	00012300
000265	000	C-----	00012400
000266	000	900 CONTINUE	00012500
000267	000	IF(NAPC.NE.0) CALL PTLOAD(NAPC,ULOAD)	00012600
000268	000	901 IF(NULOAD.NE.0) CALL EGLOAD(ULOAD)	00012700
000269	000	C	00012800
000270	000	SCAL = NINCR	00012900
000271	000	DO 200 I = 1,NFREE	00013000
000272	000	200 APF(I) = APF(I) / SCAL	00013100
000273	000	FEL = 0.	00013200
000274	000	FINC = ULOAD / SCAL	00013300
000275	000	C	00013400
000276	000	500 FORMAT(10I5)	00013500
000277	000	501 FORMAT(//, NUMBER OF NODES =',I5/', NUMBER OF ELEMENTS =',I00013600	
000278	000	15/', NUMBER OF APLIED CONCENTRATED LOADS =',I5/', NUMBER OF	00013700
000279	000	2INCREMENTAL LOAD STEPS =',I5/', NUMBER OF ITERATIONS PER EACH	00013800
000280	000	3INCREMENTAL LOADING =',I5/)	00013900
000281	000	510 FORMAT(20A4)	00014000
000282	000	511 FORMAT(4F20.5)	00014100
000283	000	520 FORMAT(2F10.4,2I5)	00014200

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000284 000 530 FORMAT(8F10.4) 00014300
000285 000 531 FORMAT(/' SINE PHI =',F10.4,' SMALLK =',F10.4,' KI =',00014400
000286 000 *F10.4,' PORCUSITY =',F10.4,' BETA =',F10.4,' OVERBURD00014500
000287 000 *EN PRESSURE =',F10.4,' DESIRED ACCURACY =',F10.4,' PERCENT./00014600
000288 000 * ' DEPTH OF SOIL MEDIA =',F10.4//) 00014700
000289 000 540 FORMAT(4I5) 00014800
000290 000 600 FORMAT(1H1,20X,20A4//30X,'COORDINATE VALUES'//T11,'NODE',T30,'Z-C00014900
000291 000 *OORD',TSU,'R-COORD',T65,'0,IF FREE TO Z',3X,'0,IF FREE TO R'//) 00015000
000292 000 620 FORMAT(T10,I5,T25,F10.4,T45,F10.4,T65,2(I8,5X)) 00015100
000293 000 630 FORMAT(/' YIELD STRESS =',E12.5,' DELL =',E12.5//) 00015200
000294 000 640 FORMAT(/' IMAX,IHB,LT,LAST',4I10//) 00015300
000295 000 650 FORMAT(5I7) 00015400
000296 000 651 FORMAT(1H1,10X,'CONNECTIVITY'//) 00015500
000297 000 RETURN 00015600
000298 000 END 00015700
000299 000 WELT,SIH NASA*IPFS,DMATRX,,,132671133010
000300 000 SUBROUTINE DMATRX(NI) 00000100
000301 000 C----- 00000200
000302 000 PARAMETER NOUT=300,NELS=260,NF=20000,MAX=600 00000300
000303 000 C----- 00000400
000304 000 COMMON /BLK0/ TITLE(20),INODE,NELEM,NAPC,NBC,NINCR,IPRINT,EPSLON 00000500
000305 000 COMMON /BLK1/ W(6),H(6),AR(4),BR(4),CR(4),AZ(4),BZ(4),CZ(4), 00000600
000306 000 * BN(4),CN(4),DN(4),TYPEA(4,4),TYPEB(4,4),TYPEC(4,4),TYPEE(4,4), 00000700
000307 000 * TYPEF(4,4),YPEG(4,4),AO,BO,CO,RT,RB,RA,RC,IC,JC,KC,LC,NEL 00000800
000308 000 COMMON /BLK4/ STRS(NELS,4),DMAT(NELS,4,4),DELMN1(MAX),POP 00000900
000309 000 COMMON /BLK5/ DE(NELS,4,4),SIGRA(NELS),DSIGRA(NELS),DELL,YSTRS, 00001000
000310 000 1 FINC,FN,ULOAD,FEL,PMAX,DLMAX 00001100
000311 000 COMMON /BLK6/ SIGR,SIGZ,SIGT,TAUZR,D(4,4),STIFF(8,8),KK(NELS,8), 00001200
000312 000 1 R(NODS),Z(NODS),TUTDIS(MAX) 00001300
000313 000 COMMON /BLK7/ USTRS(NELS,4),DUM(NELS,10) 00001400
000314 000 COMMON/BLK8/INCR,PDEPTH(NELS),VOIDI,ALAMDA,DEPTH,XMS 00001500
000315 000 COMMON /BLK9/ PI,SMALLK,CK,BETA,PO,NFREE,NELST,ICASE,NRIGD 00001600
000316 000 COMMON /BLK11/ VOID(NELS),DGAM(NELS,4) 00001700
000317 000 COMMON/BLK12/ SIGMX(NELS),DEP(NELS),EP(NELS),DEQST2(NELS),ES 00001800
000318 000 DIMENSION RLR(4),DR(4),RD(4) 00001900
000319 000 C----- 00002000
000320 000 VOIDR = VOID(NEL) 00002100
000321 000 P = (SIGZ + SIGT + SIGR) / 3. 00002200
000322 000 TJ = ((SIGZ-SIGR)**2+(SIGR-SIGT)**2+(SIGT-SIGZ)**2)/6.+TAUZR**2 00002300
000323 000 PSQ = P*P 00002400
000324 000 ETS = 3.*TJ/PSQ 00002500
000325 000 POW = 1.-SMALLK/ALAMDA 00002600
000326 000 SIGMX(NEL) = P * ((XMS+ETS) /XMS) ** POW 00002700
000327 000 POP = SIGMX(NEL) 00002800
000328 000 AA = XMS * (2.*P-POP) / 3. 00002900
000329 000 BB = 3. * AA * XMS * P * POP * (1.+VOIDR) / BETA 00003000
000330 000 DO 100 I = 1,4 00003100
000331 000 DO 100 J = 1,4 00003200
000332 000 100 D(I,J) = DE(NEL,I,J) 00003300
000333 000 SZZ = 2.*SIGZ-SIGR-SIGT 00003400
000334 000 SRR = 2.*SIGR-SIGZ-SIGT 00003500
000335 000 STT = 2.*SIGT-SIGZ-SIGR 00003600
000336 000 SZR = 6.*TAUZR 00003700
000337 000 RLB(1) = SZZ + AA 00003800
000338 000 RLB(2) = SRR + AA 00003900
000339 000 RLB(3) = STT + AA 00004000
000340 000 RLB(4) = SZR 00004100

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000341	000	CELP = (USTRS(NEL,1)+DSTRS(NEL,2)+DSTRS(NEL,3)) / 3.	00004200
000342	000	DB(1) = SZZ	00004300
000343	000	DB(2) = SRR	00004400
000344	000	DB(3) = STT	00004500
000345	000	DB(4) = SZR	00004600
000346	000	DF1 = 0.	00004700
000347	000	DFK = 0.	00004800
000348	000	DO 200 I = 1,4	00004900
000349	000	DF1 = DF1 + RLB(1) * DSTRS(NEL,I)	00005000
000350	000	200 DFK = DFK + DB(I) * DSTRS(NEL,I)	00005100
000351	000	DFJ = DELP * POP * XMS	00005200
000352	000	POW = -SMALLK / ALAMDA	00005300
000353	000	DEN = (1.+ETS/XMS) ** POW	00005400
000354	000	DFK = DEN * (DFK-6.*TJ/P*DELP)	00005500
000355	000	DF = DFI - DFJ - DFK * (1.-SMALLK/ALAMDA)	00005600
000356	000	ASQ = XMS * P * (POP-P)	00005700
000357	000	TTJ = TJ * 3.	00005800
000358	000	SIGBA(NEL) = ASQ	00005900
000359	000	EP(NEL) = DF	00006000
000360	000	WRITE(6,620) NEL,VOIDR,TTJ,ASQ,POP,DF,XMS,ETS	00006100
000361	000	620 FORMAT(15,' VOIDR=',F10.4,' 3J=',F10.4,' ASQ=',F10.4,' PO=',	00006200
000362	000	IF10.4,' DF=',E12.5,' XMS=',E12.5,' ETS=',E12.5)	00006300
000363	000	C	00006400
000364	000	IF(DF.LT.0.) GO TO 764	00006500
000365	000	C	00006600
000366	000	DO 110 I = 1,4	00006700
000367	000	DB(I) = 0.	00006800
000368	000	RD(I) = 0.	00006900
000369	000	DO 110 J = 1,4	00007000
000370	000	DB(I) = DB(I) + D(1,J) * RLB(J)	00007100
000371	000	110 RD(I) = RD(I) + RLB(J) * D(J,I)	00007200
000372	000	DEN = 0.	00007300
000373	000	DO 120 I = 1,4	00007400
000374	000	120 DEN = DEN + RLB(1) * DB(1)	00007500
000375	000	DEN = DEN + BR	00007600
000376	000	DO 130 I = 1,4	00007700
000377	000	DO 130 J = 1,4	00007800
000378	000	130 D(1,J) = D(I,J) - DB(I) * RD(J) / DEN	00007900
000379	000	764 DO 111 I = 1,4	00008000
000380	000	DO 111 J = 1,4	00008100
000381	000	111 DMAT(NEL,I,J) = D(I,J)	00008200
000382	000	600 FORMAT(4E20.7)	00008300
000383	000	RETURN	00008400
000384	000	END	00008500
000385	000	WELT,SIH NASA*TPF\$.STIFF1,,113762121110	
000386	000	SUBROUTINE STIFF1(NTAPE)	00000100
000387	000	C-----	00000200
000388	000	PARAMETER NODS=300,NELS=260,NF=20000,MAX=600	00000300
000389	000	C-----	00000400
000390	000	COMMON /BLK0/ TITLE(20),INODE,NELEM,NAPC,NBC,NINCR,IPRINT,EPSLON	00000500
000391	000	COMMON /BLK1/ W(6),H(6),AR(4),BR(4),CR(4),AZ(4),BZ(4),CZ(4),	00000600
000392	000	* BN(4),CN(4),DN(4),TYPEA(4,4),TYPEB(4,4),TYPEC(4,4),TYPEE(4,4),	00000700
000393	000	* TYPEF(4,4),TYPEG(4,4),AO,BO,CO,RT,RB,RA,RC,IC,JC,KC,LC,NEL	00000800
000394	000	COMMON /BLK6/ SIGR,SIGZ,SIGT,TAUZR,D(4,4),STIFF(8,8),KK(NELS,8),	00000900
000395	000	1 R(NODS),Z(NODS),IUTDIS(MAX)	00001000
000396	000	COMMON /BLK7/ DSTRS(NELS,4),ARM(NELS,4),AZM(NELS,4),RTT(NELS),	00001100
000397	000	1 AOJ(NELS)	00001200

000398	000	COMMON /BLKA/M,N	00001300
000399	000	C-----	00001400
000400	000	C	00001500
000401	000	C	00001600
000402	000	C	00001700
000403	000	C	00001800
000404	000	AZ(1) = Z(LC) - Z(JC)	00001900
000405	000	AZ(2) = Z(IC) - Z(KC)	00002000
000406	000	AZ(3) = -AZ(1)	00002100
000407	000	AZ(4) = -AZ(2)	00002200
000408	000	BZ(1) = Z(KC) - Z(LC)	00002300
000409	000	BZ(2) = -BZ(1)	00002400
000410	000	BZ(3) = Z(JC) - Z(IC)	00002500
000411	000	BZ(4) = -BZ(3)	00002600
000412	000	CZ(1) = Z(KC) - Z(JC)	00002700
000413	000	CZ(2) = Z(IC) - Z(LC)	00002800
000414	000	CZ(3) = -CZ(2)	00002900
000415	000	CZ(4) = -CZ(1)	00003000
000416	000	AR(1) = R(JC) - R(LC)	00003100
000417	000	AR(2) = R(KC) - R(IC)	00003200
000418	000	AR(3) = -AR(1)	00003300
000419	000	AR(4) = -AR(2)	00003400
000420	000	BR(1) = R(LC) - R(KC)	00003500
000421	000	BR(2) = -BR(1)	00003600
000422	000	BR(3) = R(IC) - R(JC)	00003700
000423	000	BR(4) = -BR(3)	00003800
000424	000	CR(1) = R(JC) - R(KC)	00003900
000425	000	CR(2) = R(LC) - R(IC)	00004000
000426	000	CR(3) = -CR(2)	00004100
000427	000	CR(4) = -CR(1)	00004200
000428	000	A0 = -AR(3)*AZ(2) + AR(4)*AZ(1)	00004300
000429	000	B0 = -BR(2)*BZ(4) + BR(3)*BZ(1)	00004400
000430	000	C0 = CR(3)*CZ(1) - CR(4)*CZ(2)	00004500
000431	000	C	00004600
000432	000	RT = R(IC) + R(JC) + R(KC) + R(LC)	00004700
000433	000	RA = -R(KC) + R(IC) - R(LC) + R(JC)	00004800
000434	000	RB = R(JC) - R(IC) + R(KC) - R(LC)	00004900
000435	000	RC = -R(IC) + R(JC) - R(KC) + R(LC)	00005000
000436	000	A0J(NEL) = A0	00005100
000437	000	RTI(NEL) = RT	00005200
000438	000	C	00005300
000439	000	DO 200 M = 1,4	00005400
000440	000	ARM(NEL,M) = AR(M)	00005500
000441	000	AZM(NEL,M) = AZ(M)	00005600
000442	000	DO 200 N = 1,4	00005700
000443	000	CALL GAUSS(1,AA)	00005800
000444	000	CALL GAUSS(2,BB)	00005900
000445	000	CALL GAUSS(3,CC)	00006000
000446	000	CALL GAUSS(4,EE)	00006100
000447	000	CALL GAUSS(5,FF)	00006200
000448	000	CALL GAUSS(6,GG)	00006300
000449	000	TYPEA(M,N) = AA	00006400
000450	000	TYPEB(M,N) = BB	00006500
000451	000	TYPEC(M,N) = CC	00006600
000452	000	TYPEE(M,N) = EE	00006700
000453	000	TYPEF(M,N) = FF	00006800
000454	000	TYPEG(M,N) = GG	00006900

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000455      000      200 CONTINUE
000456      000      WRITE(NTAPE) TYPEA,TYPEB,TYPEC,      TYPEE,TYPEF,TYPEG      00007000
000457      000      RETURN      00007100
000458      000      END      00007200
000459      000      WELT,SIH NASA*TPFS,STIFF2,,,113771121110      00007300
000460      000      SUBROUTINE STIFF2(NI,NTAPE)      00000100
000461      000      C-----      00000200
000462      000      PARAMETER NODS=300,NELS=260,NF=20000,MAX=600      00000300
000463      000      C-----      00000400
000464      000      COMMON /BLK0/ TITLE(20),INODE,NELEM,NAPC,NBC,NINCR,IPRINT,EPSLON      00000500
000465      000      COMMON /BLK1/ W(6),H(6),AR(4),BR(4),CR(4),AZ(4),BZ(4),CZ(4),      00000600
000466      000      * BN(4),CN(4),DN(4),TYPEA(4,4),TYPEB(4,4),TYPEC(4,4),TYPEE(4,4),      00000700
000467      000      * TYPEF(4,4),TYPEG(4,4),AO,B0,CO,RT,RB,RA,RC,IC,JC,KC,LC,NEL      00000800
000468      000      COMMON /BLK6/ SIGR,SIGZ,SIGT,TAUZR,D(4,4),STIFF(8,8),KK(NELS,8),      00000900
000469      000      1 R(NODS),Z(NODS),TOTDIS(MAX)      00001000
000470      000      C-----      00001100
000471      000      C      00001200
000472      000      C      FORM STIFFNESS MATRIX.      00001300
000473      000      C      00001400
000474      000      IF(NI.NE.0)      00001500
000475      000      1 READ(NTAPE) TYPEA,TYPEB,TYPEC,      TYPEE,TYPEF,TYPEG      00001600
000476      000      DO 200 I = 1,4      00001700
000477      000      DO 200 J = 1,4      00001800
000478      000      C      00001900
000479      000      STIFF(I,J) = TYPEA(I,J)*D(1,1)/8.+TYPEB(I,J)*D(4,1)/ 8.+      00002000
000480      000      1 TYPEC(J,I)*D(1,4)/ 8.+TYPEE(I,J)*D(4,4)/ 8.      00002100
000481      000      STIFF(J+4,I) = TYPEB(J,I)*D(2,1)/8.+TYPEC(J,I)*D(3,1)+TYPEA(J,I)*      00002200
000482      000      1 D(4,1)/ 8.+TYPEE(J,I)*D(2,4)/ 8.+TYPEF(J,I)*D(3,4)*2+TYPEB(I,J)*      00002300
000483      000      2 D(4,4)/ 8.      00002400
000484      000      STIFF(I,J+4) = STIFF(J+4,I)      00002500
000485      000      STIFF(I+4,J+4) = TYPEE(I,J)*D(2,2)/8.+(TYPEB(I,J)+TYPEB(J,I))*      00002600
000486      000      1 D(2,4)/ 8.+TYPEA(I,J)*D(4,4)/ 8.+2.*(TYPEF(I,J)+TYPEF(J,I))*      00002700
000487      000      2 D(2,3)+(TYPEC(I,J)+TYPEC(J,I))*D(3,4) +TYPEG(I,J)*D(3,3)      00002800
000488      000      200 CONTINUE      00002900
000489      000      RETURN      00003000
000490      000      END      00003100
000491      000      WELT,SIH NASA*TPFS,FRIC1N,,,132675133010
000492      000      SUBROUTINE FRIC1N(IC,KC,NEL,NFT,ISHEAR,VOIDI)      00000100
000493      000      PARAMETER NODS=300,NELS=260,NF=20000,MAX=600      00000200
000494      000      COMMON /BLK2/ ID(NODS,2),IJKL(NELS,4),DE1,DE2      00000300
000495      000      COMMON /BLK4/ STRS(NELS,4),DMAT(NELS,4,4),DELNM1(MAX),POP      00000400
000496      000      COMMON /BLK5/ DE(NELS,4,4),SIGBA(NELS),DSIGBA(NELS),DELL,YSTRS,      00000500
000497      000      1 F1NC,FN,ULOAD,FEE,PMAX,DLMAX      00000600
000498      000      COMMON /BLK6/ SIGR,SIGZ,SIGT,TAUZR,D(4,4),STIFF(8,8),KK(NELS,8),      00000700
000499      000      1 R(NODS),Z(NODS),TOTDIS(MAX)      00000800
000500      000      COMMON /BLK9/ PI,SMALLK,CK,BETA,PO,NFREE,NELST,ICASE,NRIGD      00000900
000501      000      COMMON /BLK10/ FRCK(20,8,8),TR(20,8,8),XXL(20),DZZ(20),DRR(20),      00001000
000502      000      1 POR(20),DELTS,EC,XNUC,DELD      00001100
000503      000      DIMENSION TS(8,8)      00001200
000504      000      DZ = DZZ(NFT)      00001300
000505      000      DR = DRR(NFT)      00001400
000506      000      PPI = 3.14159      00001500
000507      000      RB = (R(IC)+R(KC)) / 2.      00001600
000508      000      PIR = PPI * RB / 3.      00001700
000509      000      BASE = R(KC) - R(IC)      00001800
000510      000      HIGH = Z(IC) - Z(KC)      00001900
000511      000      XL = (BASE*BASE + HIGH*HIGH)**.5      00002000

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000512	000	XXL(NFT) = XL	00002100
000513	000	SINT = BASE / XL	00002200
000514	000	COST = HIGH / XL	00002300
000515	000	WRITE(6,600) NEL,XL,SINT,COST	00002400
000516	000	IF (ISHEAR.NE.0) GO TO 160	00002500
000517	000	600 FORMAT(' NEL,XL,SINT,COST',I5,3F12.4)	00002600
000518	000	DO 120 I = 1,8	00002700
000519	000	120 TR(NFT,I,I) = COST	00002800
000520	000	DO 130 J = 1,4	00002900
000521	000	J = 1 + 4	00003000
000522	000	TR(NFT,I,J) = -SINT	00003100
000523	000	130 TR(NFT,J,I) = SINT	00003200
000524	000	DII = DZ	00003300
000525	000	DIJ = 0.	00003400
000526	000	DJJ = DR	00003500
000527	000	GO TO 170	00003600
000528	000	160 CONTINUE	00003700
000529	000	NAMELIST/NAME1/ SIGZZ,SIGRR,TAURZ,SIGN,SHEAR,TAUF,DII,DZ	00003800
000530	000	II = IO(NFT,1)	00003900
000531	000	JJ = IO(NFT,2)	00004000
000532	000	SIGZZ = (STRS(II,1)+STRS(JJ,1)) / 2.	00004100
000533	000	SIGRR = (STRS(II,2)+STRS(JJ,2)) / 2.	00004200
000534	000	TAURZ = (STRS(II,4)+STRS(JJ,4)) / 2.	00004300
000535	000	SIGN = SIGZZ*COST+COST*SIGRR*SINT*SINT-TAURZ*COST*SINT	00004400
000536	000	SHEAR = ABS(STRS(NEL,1))	00004500
000537	000	TAUF = CK + SIGN*DELTS	00004600
000538	000	DII = DZ * (1.-SHEAR/TAUF)	00004700
000539	000	DZZ(NFT) = DII	00004800
000540	000	DIJ = 0.	00004900
000541	000	WRITE(6,NAME1)	00005000
000542	000	170 CONTINUE	00005100
000543	000	DIIF = DII * PIR / XXL(NFT)	00005200
000544	000	DIJF = DIJ * PIR / XXL(NFT)	00005300
000545	000	DJJF = DJJ * PIR / XXL(NFT)	00005400
000546	000	DO 190 I = 1,4	00005500
000547	000	STIFF(I,1) = 2.*DIIF	00005600
000548	000	STIFF(I+4,I+4) = 2.*DIJF	00005700
000549	000	STIFF(I,1+4) = 2.*DIJF	00005800
000550	000	190 STIFF(I,9-1) = DIJF	00005900
000551	000	DO 191 I = 1,2	00006000
000552	000	STIFF(I,1+2) = -DIIF	00006100
000553	000	STIFF(I,5-1) = DIIF	00006200
000554	000	STIFF(I,7-1) = -2.*DIJF	00006300
000555	000	STIFF(I,1+6) = -DIJF	00006400
000556	000	STIFF(I+4,I+6) = -DIJF	00006500
000557	000	STIFF(I+4,9-1) = DIJF	00006600
000558	000	STIFF(I+2,1+4) = -DIJF	00006700
000559	000	STIFF(I+2,9-1) = -2.*DIJF	00006800
000560	000	STIFF(2+1-1,2+1) = -2.*DIIF	00006900
000561	000	191 STIFF(2+1+3,2+1+4) = -2.*DIJF	00007000
000562	000	DO 210 I = 1,8	00007100
000563	000	DO 210 J = 1,8	00007200
000564	000	210 STIFF(J,I) = STIFF(I,J)	00007300
000565	000	DMAT(NEL,1,1) = DII	00007400
000566	000	DMAT(NEL,1,2) = DIJ	00007500
000567	000	DMAT(NEL,2,2) = DJJ	00007600
000568	000	WRITE(6,630) NEL,DZ,DR,DII,DIJ,DJJ,XXL(NFT)	00007700

000569	000	630 FORMAT(' NEL DZ DR DII DIJ DIJ LENGTH',I5,6E12.5)	00007800
000570	000	WRITE(6,610) STIFF	00007900
000571	000	610 FORMAT(8E15,6)	00008000
000572	000	DO 140 I = 1,8	00008100
000573	000	DO 140 J = 1,8	00008200
000574	000	SUM = 0.	00008300
000575	000	DO 144 K = 1,8	00008400
000576	000	144 SUM = SUM + TR(NFT,K,I) * STIFF(K,J)	00008500
000577	000	140 TS(I,J) = SUM	00008600
000578	000	DO 150 I = 1,8	00008700
000579	000	DO 150 J = 1,8	00008800
000580	000	SUM = 0.	00008900
000581	000	DO 155 K = 1,8	00009000
000582	000	155 SUM = SUM + TS(I,K) * TR(NFT,K,J)	00009100
000583	000	150 STIFF(I,J) = SUM	00009200
000584	000	DO 200 I = 1,8	00009300
000585	000	DO 200 J = 1,8	00009400
000586	000	200 FRCK(NFT,I,J) = STIFF(I,J)	00009500
000587	000	WRITE(6,610) STIFF	00009600
000588	000	RETURN	00009700
000589	000	END	00009800
000590	000	WELT,SIH NASA*TPFS.ASSEMB,,,114006121110	
000591	000	SUBROUTINE ASSEMB(NEL,NFT)	00000100
000592	000	C-----	00000200
000593	000	PARAMETER NODS=300,NELS=260,NF=20000,MAX=600	00000300
000594	000	C-----	00000400
000595	000	COMMON /BLK0/ TITLE(20),INODE,NELEM,NAPC,NBC,NINCR,IPRINT,EPSLON	00000500
000596	000	COMMON /BLK2/ ID(NODS,2),IJKL(NELS,4),DE1,DE2	00000600
000597	000	COMMON /BLK3/ XK(NF),APF(MAX),IMAX,IHB,IHBI,LT,LAST	00000700
000598	000	COMMON /BLK6/ SIGR,SIGZ,SIGT,TAUZR,D(4,4),STIFF(8,8),KK(NELS,8),	00000800
000599	000	1 R(NODS),Z(NODS),TOTDIS(MAX)	00000900
000600	000	COMMON /BLK10/ FRCK(20,8,8),TR(20,8,8),XXL(20),DZZ(20),DRR(20),	00001000
000601	000	1 POR(20),DELTS,EC,XNUC,DELD	00001100
000602	000	C-----	00001200
000603	000	C	00001300
000604	000	NFREE = INODE * 2	00001400
000605	000	IF(NFT.LT.1.OR.NF1.GT.NBC) GO TO 120	00001500
000606	000	DO 130 I = 1,8	00001600
000607	000	DO 130 J = 1,8	00001700
000608	000	130 STIFF(I,J) = FRCK(NFT,I,J)	00001800
000609	000	120 CONTINUE	00001900
000610	000	C	00002000
000611	000	DO 110 I = 1,8	00002100
000612	000	II = KK(NEL,I)	00002200
000613	000	DO 110 J = 1,8	00002300
000614	000	JJ = KK(NEL,J)	00002400
000615	000	IF(I1.EQ.0.OR.JJ.EQ.0) GO TO 110	00002500
000616	000	IF(I1.LT.JJ) GO TO 110	00002600
000617	000	IF(I1.GT.IHBI) GO TO 104	00002700
000618	000	L = JJ + (II-1) * II / 2	00002800
000619	000	GO TO 105	00002900
000620	000	104 L = JJ + LT + (II-IHB) * IHBI	00003000
000621	000	105 XK(L) = -XK(L) + STIFF(I,J)	00003100
000622	000	110 CONTINUE	00003200
000623	000	RETURN	00003300
000624	000	END	00003400
000625	000	WELT,SIH NASA*TPFS.DISPL,,,11401121110	

000626	000	SUBROUTINE DISPL(NFREE,NI,INODE,INCR)	00000100
000627	000	C-----	00000200
000628	000	PARAMETER NODS=300,NELS=260,NF=20000,MAX=600	00000300
000629	000	C-----	00000400
000630	000	COMMON /BLK2/ ID(NODS,2),IJKL(NELS,4),DE1,DE2	00000500
000631	000	COMMON /BLK3/ XK(NF),APF(MAX),IMAX,IHB,IHBI,LT,LAST	00000600
000632	000	COMMON /BLK6/ SIGR,SIGZ,SIGT,TAUZR,D(4,4),STIFF(8,8),KK(NELS,8),	00000700
000633	000	1 R(NODS),Z(NODS),TUTDIS(MAX)	00000800
000634	000	C-----	00000900
000635	000	C	00001000
000636	000	N = NFREE	00001100
000637	000	ZTEST = 0.000001	00001200
000638	000	C	00001300
000639	000	DO 100 J = 1,NFREE	00001400
000640	000	IF(J.GT.IHBI) GO TO 108	00001500
000641	000	L = (J+1) * J / 2	00001600
000642	000	GO TO 109	00001700
000643	000	108 L = LT + IHB * (J - IHBI)	00001800
000644	000	109 XTEST = ABS(XK(L))	00001900
000645	000	IF(XTEST.LT.ZTEST) XK(L) = 1.	00002000
000646	000	100. CONTINUE	00002100
000647	000	C	00002200
000648	000	DO 110 I = 1,NFREE	00002300
000649	000	110 XK(LAST+1) = APF(I)	00002400
000650	000	C	00002500
000651	000	C	00002600
000652	000	CALL FACTOR(NFREE)	00002700
000653	000	C	00002800
000654	000	C	00002900
000655	000	CALL SOLTN(NFREE)	00003000
000656	000	C	00003100
000657	000	C	00003200
000658	000	WRITE(6,600) INCR,NI	00003300
000659	000	NN = 10	00003400
000660	000	DO 280 J = 1,NN	00003500
000661	000	II = (J-1) * 2 + 1 + LAST	00003600
000662	000	JJ = II + 1	00003700
000663	000	280 WRITE(6,610) J,XK(II),XK(JJ)	00003800
000664	000	600 FORMAT(/// 20X,'DISPLACEMENTS FOR CYCLE NO.',I4//18X,'Z - DISPL',	00003900
000665	000	* 15X,'R - DISPL',15X,'LOAD INCREMENT STEP =',I5//)	00004000
000666	000	610 FORMAT(I7,2E20.7)	00004100
000667	000	RETURN	00004200
000668	000	END	00004300
000669	000	WELT,SIH NASA*TPFS,FACTOR,,,114013121110	
000670	000	SUBROUTINE FACTOR(NFREE)	00000100
000671	000	C THIS SUBROUTINE PERFORMS FACTORING	00000200
000672	000	PARAMETER NODS=300,NELS=260,NF=20000,MAX=600	00000300
000673	000	COMMON /BLK3/ XK(NF),APF(MAX),IMAX,IHB,IHBI,LT,LAST	00000400
000674	000	N = NFREE	00000500
000675	000	IHB1 = IHB1	00000600
000676	000	DO 8 J=1,N	00000700
000677	000	IF(I.GT.IHB1) GO TO 2	00000800
000678	000	K=1	00000900
000679	000	M=K+(I-1)*1/2	00001000
000680	000	GO TO 3	00001100
000681	000	2 K=I-IHB1	00001200
000682	000	M=K+LT+(I-IHB)*IHB1	00001300

000683	000	3 J=I+IHB1	00001400
000684	000	IF(J.GT.N) GO TO 4	00001500
000685	000	JJ=I+IHB1	00001600
000686	000	GO TO 5	00001700
000687	000	4 JJ=N	00001800
000688	000	5 B=0.0	00001900
000689	000	LA=I-1	00002000
000690	000	LB=I+1	00002100
000691	000	IF(LA.EQ.0) GO TO 6	00002200
000692	000	DO 7 L=K,LA	00002300
000693	000	IF(L.GT.IHB1)GO TO 50	00002400
000694	000	J = (L+1)*L/2	00002500
000695	000	GO TO 51	00002600
000696	000	50 J = LT + IHB*(L-IHB1)	00002700
000697	000	51 A = XK(M)	00002800
000698	000	R = B+A*A*XK(J)	00002900
000699	000	7 M=M+1	00003000
000700	000	6 A=XK(M)	00003100
000701	000	XK(M)=A-B	00003200
000702	000	IF(I.EQ.N) GO TO 8	00003300
000703	000	DO 9 J=LB,JJ	00003400
000704	000	SUM=0.0	00003500
000705	000	IF(J.GT.IHB1) GO TO 10	00003600
000706	000	K=1	00003700
000707	000	MM=K+(J-1)*J/2	00003800
000708	000	GO TO 11	00003900
000709	000	10 K=J-IHB1	00004000
000710	000	MM=K+LT+(J-IHB)*IHB1	00004100
000711	000	11 IF(LA.EQ.0) GO TO 9	00004200
000712	000	IF(K.GT.LA) GO TO 9	00004300
000713	000	DO 12 JA=K,LA	00004400
000714	000	L=M-1+JA	00004500
000715	000	IF(JA.GT.IHB1) GO TO 13	00004600
000716	000	L1=(JA+1)*JA/2	00004700
000717	000	GO TO 14	00004800
000718	000	13 L1=LT+IHB*(JA-IHB1)	00004900
000719	000	14 SUM=SUM+XK(MM)*XK(L)*XK(L1)	00005000
000720	000	12 MM=MM+1	00005100
000721	000	9 XK(MM)=(XK(MM)-SUM)/XK(M)	00005200
000722	000	8 CONTINUE	00005300
000723	000	RETURN	00005400
000724	000	END	00005500
000725	000	WELT,SIH NASA*TPFS,SOLTN,,,114016121110	
000726	000	SUBROUTINE SOLTN(NFREE)	00000100
000727	000	PARAMETER NOUS=300,NELS=260,NT=20000,MAX=600	00000200
000728	000	COMMON /BLK3/ XK(NT),APF(MAX),IMAX,IHB,IHB1,LT,LAST	00000300
000729	000	C THIS PORTION OF SUBROUTINE PERFORMS FORWARD-SUBSTITUTION	00000400
000730	000	C	00000500
000731	000	N = NFREE	00000600
000732	000	IHB1 = IHB	00000700
000733	000	NF = LAST + 1	00000800
000734	000	C	00000900
000735	000	14 DO 1 K = 2,N	00001000
000736	000	C	00001100
000737	000	IF(K.GT.IHB1) GO TO 2	00001200
000738	000	M=0	00001300
000739	000	MM=K-1	00001400

000740	000	M1=MM*K/2	00001500
000741	000	GO TO 3	00001600
000742	000	2 M=N-IHB	00001700
000743	000	MM=IHB1	00001800
000744	000	M1=M*IHB1+LT	00001900
000745	000	3 SUM=0.0	00002000
000746	000	DO 4 L=1,MM	00002100
000747	000	LL=L+M	00002200
000748	000	JJ=LL+M1	00002300
000749	000	LL=LL+NF-1	00002400
000750	000	4 SUM=SUM+XK(JJ)*XK(LL)	00002500
000751	000	1 XK(LL+1)=XK(LL+1)-SUM	00002600
000752	000	J = NF+N-1	00002700
000753	000	C THIS PORTION OF SUBROUTINE PERFORMS BACK-SUBSTITUTION	00002800
000754	000	NF=NF+N-1	00002900
000755	000	XK(NF)=XK(NF)/XK(LAST)	00003000
000756	000	DO 5 K=2,N	00003100
000757	000	L=N-K+1	00003200
000758	000	IF(L.GT.IHB1) GO TO 6	00003300
000759	000	I=L+(L-1)*L/2	00003400
000760	000	GO TO 7	00003500
000761	000	6 I=L+(L-IHB)*IHB1+LT	00003600
000762	000	7 IR=N-IHB	00003700
000763	000	IF(L.GT.IR) GO TO 8	00003800
000764	000	J=IHB1	00003900
000765	000	GO TO 9	00004000
000766	000	8 J=K-1	00004100
000767	000	9 SUM=0.0	00004200
000768	000	DO 10 M=1,J	00004300
000769	000	MM=L+M	00004400
000770	000	IF(MM.GT.IHB1) GO TO 11	00004500
000771	000	NN=L+(MM-1)*MM/2	00004600
000772	000	GO TO 12	00004700
000773	000	11 NN=L+(MM-IHR)*IHB1+LT	00004800
000774	000	12 MM=NF-N+MM	00004900
000775	000	10 SUM=SUM+XK(NN)*XK(MM)	00005000
000776	000	MM=NF-N+L	00005100
000777	000	5 XK(MM)=XK(MM)/XK(1)-SUM	00005200
000778	000	RETURN	00005300
000779	000	END	00005400
000780	000	WELT,SIH NASA*TPFS,STRAIN,,,132705133010	
000781	000	SUBROUTINE STRAIN(NI,INCR)	00000100
000782	000	C-----	00000200
000783	000	PARAMETER NODS=300,NELS=260,NF=20000,MAX=600	00000300
000784	000	C-----	00000400
000785	000	COMMON /BLK0/ TITLE(20),INODE,NELEM,NAPC,NBC,NINCR,IPRINT,EPSLON	00000500
000786	000	COMMON /BLK2/ ID(NODS,2),IJKL(NELS,4),DE1,DE2	00000600
000787	000	COMMON /BLK3/ XK(NF),APF(MAX),IMAX,IHR,IHB1,LT,LAST	00000700
000788	000	COMMON /BLK4/ STRS(NELS,4),DMAT(NELS,4,4),DELMN1(MAX),POP	00000800
000789	000	COMMON /BLK5/ DE(NELS,4,4),SIGBA(NELS),DSIGBA(NELS),DELL,YSTRS,	00000900
000790	000	1 FINC,FN,ULOAD,FEL,PMAX,DLMAX	00001000
000791	000	COMMON /BLK7/ DSTHS(NELS,4),ARM(NELS,4),AZM(NELS,4),RTT(NELS),	00001100
000792	000	1 AQU(NELS)	00001200
000793	000	COMMON /BLK8/ INCNN,PDEPTH(NELS),VOIDI,ALAMDA,DEPTH,XMS	00001300
000794	000	COMMON /BLK9/ PI,SMALLK,CK,BETA,PO,NEREE,NELST,ICASE,NRIGN	00001400
000795	000	COMMON /BLK11/ VOID(NELS),UGAM(NELS,4)	00001500
000796	000	COMMON /BLK12/ SIGMX(NELS),DEP(NELS),EP(NELS),DEGST2(NELS),ES	00001600

000797	000	-DIMENSION	U(4),V(4),EPS(4),SIG(4)	00001700
000798	000	C		00001800
000799	000	C		00001900
000800	000		SUMA = 0.	00002000
000801	000		SUMB = 0.	00002100
000802	000		IF(NBC.NE.0) CALL INTFAC(DEL,N1)	00002200
000803	000		WRITE(6,666) N1,INCR	00002300
000804	000	666	FORMAT(///10X,'STRAINS AND STRESSES FOR INCREMENT NO. =',I4,	00002400
000805	000		* ' NO. OF CYCLES = ',I4,///5X,'ELEM',T14,'SIG-Z',T30,'SIG-R',T45,	00002500
000806	000		1'SIG-T',T60,'TAU-ZR',T75,'AREA',T90,'VOID RATIO')	00002600
000807	000		DO 900 NEL = 1, NELEM	00002700
000808	000		NFT = NEL - NRIGD	00002800
000809	000		IF(NFT.GT.0.AND.NFT.LE.NBC) GO TO 900	00002900
000810	000		DO 100 I = 1,4	00003000
000811	000		IN = IJKL(NEL,I)	00003100
000812	000		II = (IN-1)*2 + LAST + 1	00003200
000813	000		JJ = II + 1	00003300
000814	000		V(1) = XK(II)	00003400
000815	000	100	U(1) = XK(JJ)	00003500
000816	000	C	FIND STRAINS AND STRESSES AT THE CENTROID.	00003600
000817	000		EZ = 0.	00003700
000818	000		ER = 0.	00003800
000819	000		GM = 0.	00003900
000820	000		SUM = 0.	00004000
000821	000		DO 111 I = 1,4	00004100
000822	000		EZ = EZ + ARM(NEL,I) * V(I)	00004200
000823	000		ER = ER + AZM(NEL,I) * U(I)	00004300
000824	000		GM = GM + ARM(NEL,I) * U(I) + AZM(NEL,I) * V(I)	00004400
000825	000	111	SUM = SUM + U(I)	00004500
000826	000	C	COMPRESSION POSITIVE.	00004600
000827	000		EPS(1) = -EZ / A0J(NEL)	00004700
000828	000		EPS(2) = -ER / A0J(NEL)	00004800
000829	000		EPS(3) = -SUM / RIT(NEL)	00004900
000830	000		EPS(4) = -GM / A0J(NEL)	00005000
000831	000	C	EPS(1) = STRAIN IN Z - DIRECTION.	00005100
000832	000	C	EPS(2) = STRAIN IN R - DIRECTION.	00005200
000833	000	C	EPS(3) = TANGENTIAL STRAIN.	00005300
000834	000	C	EPS(4) = SHEAR STRAIN	00005400
000835	000		DO 200 I = 1,4	00005500
000836	000		DGAM(NEL,I) = EPS(1)	00005600
000837	000		SIG(1) = 0.	00005700
000838	000		DO 200 J = 1,4	00005800
000839	000		SIG(I) = SIG(I) + DMAT(NEL,I,J) * EPS(J)	00005900
000840	000	200	CONTINUE	00006000
000841	000		III=NEL	00006100
000842	000		DO 210 I = 1,4	00006200
000843	000		DSTRS(III,1)=SIG(1)	00006300
000844	000	210	CONTINUE	00006400
000845	000		IF(NEL.LE.NRIGD) GO TO 890	00006500
000846	000		CALL AREAA(IJKL(NEL,1),IJKL(NEL,2),IJKL(NEL,3),IJKL(NEL,4),AREA)	00006600
000847	000		RATE = AREA / DELNM1(NEL)	00006700
000848	000		VOID(NEL) = RATE * (1.+VOIDI) - 1.	00006800
000849	000	890	CONTINUE	00006900
000850	000		WRITE(6,600) NEL,SIG,AREA,VOID(NEL)	00007000
000851	000	900	CONTINUE	00007100
000852	000	C		00007200
000853	000	600	FORMAT(I9,7F15.6)	00007300

000854	000	620	FORMAT(I10,2F15.6)	00007400
000855	000	RETURN		00007500
000856	000	END		00007600
000857	000	WELT,SIH NASA*TPF\$.INTFAC,,,132711133010		
000858	000	SUBROUTINE INTFAC(GEI,NI)	00000100	
000859	000	PARAMETER NODS=300,NELS=260,NF=20000,MAX=600	00000200	
000860	000	COMMON /BLK0/ TITLE(20),INODE,NELEM,NAPC,NBC,NINCR,NCYCL, EPSLON	00000300	
000861	000	COMMON /BLK2/ ID(NODS,2),IJKL(NELS,4),DE1,DE2	00000400	
000862	000	COMMON /BLK3/ XK(NF),APF(MAX),IMAX,IHR,IHBI,LT, LAST	00000500	
000863	000	COMMON /BLK4/ STRS(NELS,4),DMAT(NELS,4,4),DELNM1(MAX),POP	00000600	
000864	000	COMMON /BLK5/ DE(NELS,4,4),SIGBA(NELS),DSIGBA(NELS),DELL,YSTRS,	00000700	
000865	000	1 FINC,FN,ULOAD,FEL,PMAX,OLMAX	00000800	
000866	000	COMMON /BLK7/ DSTRS(NELS,4),ARM(NELS,4),AZM(NELS,4),RTT(NELS),	00000900	
000867	000	1 AOJ(NELS)	00001000	
000868	000	COMMON/BLK8/INCR,PDEPTH(NELS),VOIDI,ALAMDA,DEPTH,PP	00001100	
000869	000	COMMON /BLK9/ PI,SMALLK,CK,RETA,PO,NFREE,NELST,ICASE,NRIGD	00001200	
000870	000	COMMON /BLK10/ FRCK(20,8,8),TR(20,8,8),XXL(20),DZZ(20),DRP(20),	00001300	
000871	000	1 POR(20),DELTS,EC,XNUC,DELD	00001400	
000872	000	DIMENSION UG(8),UL(8)	00001500	
000873	000	DE1 = 0.	00001600	
000874	000	DO 900 NEL = 1,NBC	00001700	
000875	000	III = NEL + NRIGD	00001800	
000876	000	DO 100 I = 1,4	00001900	
000877	000	II = IJKL(III,1) * 2 + LAST	00002000	
000878	000	UG(I) = XK(II-1)	00002100	
000879	000	100 UG(I+4) = XK(II)	00002200	
000880	000	DO 110 J = 1,8	00002300	
000881	000	UL(I) = 0.	00002400	
000882	000	DO 110 J = 1,8	00002500	
000883	000	110 UL(I) = UL(I) + TR(NEL,I,J) * UG(J)	00002600	
000884	000	WRITE(6,620) UG	00002700	
000885	000	WRITE(6,620) UL	00002800	
000886	000	EZ = (-UL(1)+UL(2)+UL(3)-UL(4)) / (2.*XXL(NEL))	00002900	
000887	000	ER = (-UL(5)+UL(6)+UL(7)-UL(8)) / (2.*XXL(NEL))	00003000	
000888	000	SIGZ = DMAT(III,1,1) * EZ + DMAT(III,1,2) * ER	00003100	
000889	000	SIGR = DMAT(III,1,2) * EZ + DMAT(III,2,2) * ER	00003200	
000890	000	SIGZ = -SIGZ	00003300	
000891	000	DSTRS(III,1) = SIGZ	00003400	
000892	000	DSTRS(III,2) = SIGR	00003500	
000893	000	NAMelist/NAME2/ I11,SIGZ,SIGR	00003600	
000894	000	WRITE(6,NAME2)	00003700	
000895	000	900 CONTINUE	00003800	
000896	000	620 FORMAT(6F15.5)	00003900	
000897	000	RETURN	00004000	
000898	000	END	00004100	
000899	000	WELT,SIH NASA*TPF\$.SETUP,,,114055121110		
000900	000	SUBROUTINE SETUP	00000100	
000901	000	COMMON /BLK1/ W(6),H(6),AR(4),BR(4),CR(4),AZ(4),BZ(4),CZ(4),	00000200	
000902	000	* HN(4),CN(4),DN(4),TYPEA(4,4),TYPEB(4,4),TYPEC(4,4),TYPEE(4,4),	00000300	
000903	000	* TYPEF(4,4),TYPEG(4,4),AO,BO,CO,RT,RB,RA,RC,IC,JC,KC,LC,NEL	00000400	
000904	000	W(1) = .1713244924	00000500	
000905	000	W(2) = .3607615730	00000600	
000906	000	W(3) = .4679139346	00000700	
000907	000	W(4) = W(3)	00000800	
000908	000	W(5) = W(2)	00000900	
000909	000	W(6) = W(1)	00001000	
000910	000	H(1) = .9324695142	00001100	

000911	000	H(2) = .6612093865	00001200
000912	000	H(3) = .2386191861	00001300
000913	000	H(4) = -H(3)	00001400
000914	000	H(5) = -H(2)	00001500
000915	000	H(6) = -H(1)	00001600
000916	000	BN(1) = -1.	00001700
000917	000	BN(2) = 1.	00001800
000918	000	BN(3) = 1.	00001900
000919	000	BN(4) = -1.	00002000
000920	000	CN(1) = 1.	00002100
000921	000	CN(2) = 1.	00002200
000922	000	CN(3) = -1.	00002300
000923	000	CN(4) = -1.	00002400
000924	000	DN(1) = -1.	00002500
000925	000	DN(2) = 1.	00002600
000926	000	DN(3) = -1.	00002700
000927	000	DN(4) = 1.	00002800
000928	000	RETURN	00002900
000929	000	END	00003000
000930	000	WELT,SIH NASA*TPFS.ELASTC,,,114060121110	
000931	000	SUBROUTINE ELASTC(D,NRIGD,E,XNU)	00000100
000932	000	DIMENSION D(4,4)	00000200
000933	000	WRITE(6,600) NRIGD,E,XNU	00000300
000934	000	CONST = E*XNU / ((1.+XNU)*(1.-XNU*2.))	00000400
000935	000	SHEAR = E / (2.*(1.+XNU))	00000500
000936	000	D(1,1) = CONST + SHEAR*2.	00000600
000937	000	D(2,2) = D(1,1)	00000700
000938	000	D(3,3) = D(1,1)	00000800
000939	000	D(4,4) = SHEAR	00000900
000940	000	D(1,2) = CONST	00001000
000941	000	D(1,3) = CONST	00001100
000942	000	D(2,3) = CONST	00001200
000943	000	DO 100 I = 1,4	00001300
000944	000	DO 100 J = 1,4	00001400
000945	000	100 D(J,I) = D(I,J)	00001500
000946	000	600 FORMAT(/' FOR FIRST',I4,' ELEMENTS, THE FOLLOWING MATERIAL PRO	00001600
000947	000	* PERTIES ARE USED TO FORM ELASTIC MATRIX (D)'/	00001700
000948	000	* ' XNU =',F10.3//)	00001800
000949	000	RETURN	00001900
000950	000	END	00002000
000951	000	WELT,SIH NASA*TPFS.GAUSS,,,114063121110	
000952	000	SUBROUTINE GAUSS(I1,AA)	00000100
000953	000	COMMON /BLK1/ W(6),H(6),AR(4),BR(4),CR(4),AZ(4),BZ(4),CZ(4),	00000200
000954	000	* BN(4),CN(4),DN(4),TYPEA(4,4),TYPEB(4,4),TYPEC(4,4),TYPEE(4,4),	00000300
000955	000	* TYPEF(4,4),TYPEG(4,4),AO,BO,CO,RT,RB,RA,RC,IC,JC,KC,LC,NEL	00000400
000956	000	TWOPI = 6.28318531	00000500
000957	000	IPT = 6	00000600
000958	000	AA = 0.	00000700
000959	000	DO 100 I = 1,IPT	00000800
000960	000	X = H(I)	00000900
000961	000	DO 100 J = 1,IPT	00001000
000962	000	Y = H(J)	00001100
000963	000	AA = AA + W(I) * W(J) * F(X,Y,I1)	00001200
000964	000	100 CONTINUE	00001300
000965	000	AA = AA * TWOPI	00001400
000966	000	RETURN	00001500
000967	000	END	00001600


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000968 000 WELT,SIH NASA*IPF$.F,,,114066121110
000969 000 FUNCTION F(X,Y,IT)
000970 000 COMMON /BLK1/ W(6),H(6),AR(4),BR(4),CR(4),A7(4),HZ(4),CZ(4),
000971 000 * RN(4),CN(4),DN(4),TYPEA(4,4),TYPEB(4,4),TYPEC(4,4),TYPEE(4,4),
000972 000 * TYPEF(4,4),TYPEG(4,4),AO,BO,CO,RT,RB,RA,RC,IC,JC,KC,LC,NFL
000973 000 COMMON /BLKA/M,N
000974 000 C X STANDS FOR ZHAI IN ZHAI - EITA COORD.
000975 000 C Y STANDS FOR EITA IN ZHAI - EITA COORD.
000976 000 C FB = DET. OF JACOBI.
000977 000 C FC = (N) * (R)
000978 000 FB = AO + BO * X + CO * Y
000979 000 FC = ( RT + RB * X + RA * Y + RC * X * Y ) / 4.
000980 000 GO TO (10,20,30,40,50,60),IT
000981 000 10 F = (AR(M)+BR(M)*A+CR(M)*Y) * (AR(N)+BR(N)*X+CR(N)*Y) / FB
000982 000 F = F * FC
000983 000 RETURN
000984 000 20 F = (AZ(M)+BZ(M)*X+CZ(M)*Y) * (AR(N)+BR(N)*X+CR(N)*Y) / FB
000985 000 F = F * FC
000986 000 RETURN
000987 000 30 F = (1.+BN(M)*X+CN(M)*Y+DN(M)*X*Y) * (AR(N)+BR(N)*X+CR(N)*Y) / 32.
000988 000 RETURN
000989 000 40 F = (AZ(M)+BZ(M)*X+CZ(M)*Y) * (AZ(N)+BZ(N)*X+CZ(N)*Y) / FB
000990 000 F = F * FC
000991 000 RETURN
000992 000 50 F = (1.+BN(M)*X+CN(M)*Y+DN(M)*X*Y) * (AZ(N)+BZ(N)*X+CZ(N)*Y) / 64.
000993 000 RETURN
000994 000 60 F = (1.+BN(M)*X+CN(M)*Y+DN(M)*X*Y) *
000995 000 1 (1.+RN(N)*X+CN(N)*Y+DN(N)*X*Y) * FB / (128.*FC)
000996 000 RETURN
000997 000 END
000998 000 WELT,SIH NASA*TPF$.AREA,,,114072121110
000999 000 SUBROUTINE AREA(IC,JC,KC,LC,AREA)
001000 000 PARAMETER NODS=300,NELS=260,NF=20000,MAX=600
001001 000 COMMON /BLK6/ SIGX,SIGZ,SIGT,TAUZR,D(4,4),STIFF(8,8),KK(NELS,8),
001002 000 1 R(NODS),Z(NODS),TUTDIS(MAX)
001003 000 AI = (R(JC)-R(IC)) * (Z(LC)-Z(IC)) - (R(LC)-R(IC)) * (Z(JC)-Z(IC))
001004 000 AJ = (R(KC)-R(JC)) * (Z(LC)-Z(JC)) - (R(LC)-R(JC)) * (Z(KC)-Z(JC))
001005 000 IF(AI.LT.0) AI = -AI
001006 000 IF(AJ.LT.0) AJ = -AJ
001007 000 AREA = (AI + AJ) / 2.
001008 000 RETURN
001009 000 END
001010 000 WELT,SIH NASA*TPF$.PTLOAD,,,114074121110
001011 000 SUBROUTINE PTLOAD(NAPC,ULOAD)
001012 000 PARAMETER NODS=300,NELS=260,NF=20000,MAX=600
001013 000 COMMON /BLK3/ XK(NF),APF(MAX),IMAX,IHR,IHBI,LT,LAST
001014 000 C
001015 000 C
001016 000 C GET CONCENTRATED LOAD IF THERE IS ANY
001017 000 WRITE(6,677)
001018 000 DO 920 NC = 1,NAPC
001019 000 READ(5,540) NODE,PZ,PK
001020 000 WRITE(6,688) NODE,PZ,PK
001021 000 II = (NODE-1) * 2
001022 000 APF(II+1) = PZ
001023 000 APF(II+2) = PK
001024 000 ULOAD = PZ

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001025      000      920 CONTINUE                                00001500
001026      000      540 FORMAT(15,2F15.6)                    00001600
001027      000      677 FORMAT(///10X,'APPLIED C. LOAD'//5X,'NODE',10X,'FORCE TO 2',5X,' 00001700
001028      000      * FORCE TO 2'//)                             00001800
001029      000      688 FORMAT(5X,15,2(5X,E12.4))             00001900
001030      000      RETURN                                       00002000
001031      000      END                                           00002100
001032      000      WELT,SIH NASA*TPFS,EGLoad,,,114076121110
001033      000      SUBROUTINE EGLoad(ULOAD)                    00000100
001034      000      C                                              00000200
001035      000      C          THIS SUBROUTINE IS TO CALCULATE THE EQUIVALENT NODAL LOADS. 00000300
001036      000      C                                              00000400
001037      000      PARAMETER N0US=300,NELS=260,NF=20000,MAX=600 00000500
001038      000      COMMON /BLK2/ ID(N0US,2),IJKL(NELS,4),DF1,DE2 00000600
001039      000      COMMON /BLK3/ XK(NF),APF(MAX),IMAX,IHR,IHBI,LT,LAST 00000700
001040      000      COMMON /BLK6/ SIGK,SIGZ,SIGT,TAUZR,D(4,4),STIFF(8,8),KK(NELS,8), 00000800
001041      000      1 R(NCDS),Z(NODS),TOTDIS(MAX)                00000900
001042      000      PI = 3.1415926                                00001000
001043      000      XXI = 0.                                        00001100
001044      000      XXJ = 0.                                        00001200
001045      000      READ(5,500) NLDEL,ULOAD                      00001300
001046      000      WRITE(6,600) NLDEL                          00001400
001047      000      DO 200 I = 1,NLDEL                          00001500
001048      000      READ(5,510) NOL,NLFT,NRHT                    00001600
001049      000      KC = NRHT                                     00001700
001050      000      LC = NLFT                                     00001800
001051      000      EGL = (R(KC)**2-R(LC)**2)*PI*ULOAD/2.        00001900
001052      000      II = (KC-1) * 2 + 1                         00002000
001053      000      JJ = (LC-1) * 2 + 1                         00002100
001054      000      APF(II) = APF(II) + EGL                      00002200
001055      000      APF(JJ) = APF(JJ) + EGL                      00002300
001056      000      200 WRITE(6,620) NOL,LC,KC,EGL,ULOAD        00002400
001057      000      620 FORMAT(16,5X,214,2F20.7)               00002500
001058      000      500 FORMAT(15,F20.9)                        00002600
001059      000      510 FORMAT(315)                             00002700
001060      000      600 FORMAT(///'      CALCULATION OF EQUIVALENT NODAL LOAD'// 00002800
001061      000      * '      NUMBER OF LOADED ELEMENTS = ',I4/ 00002900
001062      000      * '      ELEM NO----LOADED NODE-----EQUIV LOAD-----GIVEN U.LOAD'// 00003000
001063      000      610 FORMAT(2110,2F15.7)                    00003100
001064      000      RETURN                                       00003200
001065      000      END                                           00003300
001066      000      WELT,SIH NASA*TPFS,ZERO,,,114100121110
001067      000      SUBROUTINE ZERO(A,N,M)                      00000100
001068      000      DIMENSION A(1)                                00000200
001069      000      K = N * M                                       00000300
001070      000      DO 100 I = 1,K                               00000400
001071      000      100 A(I) = 0.                                  00000500
001072      000      RETURN                                       00000600
001073      000      END                                           00000700
001074      000      LMAP,IX
001075      000      EXOT
001076      000      CONE PENETROMETER
001077      000      90 71 3 100 5 2
001078      000      25.
001079      000
001080      000      7000000. 0.3 10. 0.45
001081      000      35. 0.006 0.7 0.76 0.0157 33. 0.07 73.

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001082	000	.0000	.0000	0	1	1
001083	000	.0000	.7050	0	1	2
001084	000	.0000	1.4100	0	1	3
001085	000	.0000	1.4100	0	0	4
001086	000	.0000	1.9000	0	0	5
001087	000	.0000	2.5000	0	0	6
001088	000	.0000	3.3000	0	0	7
001089	000	.0000	4.5000	0	0	8
001090	000	.0000	6.3000	0	0	9
001091	000	.0000	8.5000	0	0	10
001092	000	.0000	10.0000	0	1	11
001093	000	1.8000	.0000	0	1	12
001094	000	1.8000	.5000	0	1	13
001095	000	1.8000	.9400	0	1	14
001096	000	1.8000	.9400	0	0	15
001097	000	1.8000	1.3400	0	0	16
001098	000	1.8000	1.9400	0	0	17
001099	000	1.8000	2.7400	0	0	18
001100	000	1.8000	3.9400	0	0	19
001101	000	1.8000	5.7400	0	0	20
001102	000	1.8000	7.9400	0	0	21
001103	000	1.8000	10.0000	0	1	22
001104	000	3.6000	.0000	0	1	23
001105	000	3.4000	.2500	0	1	24
001106	000	3.6000	.4700	0	1	25
001107	000	3.6000	.4700	0	0	26
001108	000	3.6000	.8700	0	0	27
001109	000	3.6000	1.4700	0	0	28
001110	000	3.6000	2.2700	0	0	29
001111	000	3.6000	3.4700	0	0	30
001112	000	3.6000	5.2700	0	0	31
001113	000	3.6000	7.4700	0	0	32
001114	000	3.6000	10.0000	0	1	33
001115	000	5.3000	.0000	0	1	34
001116	000	5.3000	.0000	0	0	35
001117	000	5.3000	.4000	0	0	36
001118	000	5.3000	1.0000	0	0	37
001119	000	5.3000	1.8000	0	0	38
001120	000	5.3000	3.0000	0	0	39
001121	000	5.3000	4.8000	0	0	40
001122	000	5.3000	7.0000	0	0	41
001123	000	5.3000	10.0000	0	1	42
001124	000	7.4000	.0000	0	1	43
001125	000	7.4000	.4000	0	0	44
001126	000	7.4000	1.0000	0	0	45
001127	000	7.4000	1.8000	0	0	46
001128	000	7.4000	3.0000	0	0	47
001129	000	7.4000	4.8000	0	0	48
001130	000	7.4000	7.0000	0	0	49
001131	000	7.4000	10.0000	0	1	50
001132	000	10.0000	.0000	0	1	51
001133	000	10.0000	.4000	0	0	52
001134	000	10.0000	1.0000	0	0	53
001135	000	10.0000	1.8000	0	0	54
001136	000	10.0000	3.0000	0	0	55
001137	000	10.0000	4.8000	0	0	56
001138	000	10.0000	7.0000	0	0	57

001139	000	10.0000	10.0000	0	1	58
001140	000	14.0000	.0000	0	1	59
001141	000	14.0000	.4000	0	0	60
001142	000	14.0000	1.0000	0	0	61
001143	000	14.0000	1.8000	0	0	62
001144	000	14.0000	3.0000	0	0	63
001145	000	14.0000	4.8000	0	0	64
001146	000	14.0000	7.0000	0	0	65
001147	000	14.0000	10.0000	0	1	66
001148	000	19.0000	.0000	0	1	67
001149	000	19.0000	.4000	0	0	68
001150	000	19.0000	1.0000	0	0	69
001151	000	19.0000	1.8000	0	0	70
001152	000	19.0000	3.0000	0	0	71
001153	000	19.0000	4.8000	0	0	72
001154	000	19.0000	7.0000	0	0	73
001155	000	19.0000	10.0000	0	1	74
001156	000	25.0000	.0000	0	1	75
001157	000	25.0000	.4000	0	0	76
001158	000	25.0000	1.0000	0	0	77
001159	000	25.0000	1.8000	0	0	78
001160	000	25.0000	3.0000	0	0	79
001161	000	25.0000	4.8000	0	0	80
001162	000	25.0000	7.0000	0	0	81
001163	000	25.0000	10.0000	0	1	82
001164	000	33.0000	.0000	1	1	83
001165	000	33.0000	.4000	1	1	84
001166	000	33.0000	1.0000	1	1	85
001167	000	33.0000	1.8000	1	1	86
001168	000	33.0000	3.0000	1	1	87
001169	000	33.0000	4.8000	1	1	88
001170	000	33.0000	7.0000	1	1	89
001171	000	33.0000	10.0000	1	1	90
001172	000	12 13	2 1			
001173	000	13 14	3 2			
001174	000	23 24	13 12			
001175	000	24 25	14 13			
001176	000	34 25	24 23			
001177	000	14 15	4 3			
001178	000	25 26	15 14			
001179	000	34 35	26 25			
001180	000	15 16	5 4			
001181	000	26 27	16 14			
001182	000	35 36	27 26			
001183	000	16 17	6 5			
001184	000	27 28	17 16			
001185	000	36 37	28 27			
001186	000	17 18	7 6			
001187	000	28 29	28 17			
001188	000	37 38	29 28			
001189	000	18 19	8 7			
001190	000	29 30	19 18			
001191	000	38 39	30 29			
001192	000	19 20	9 8			
001193	000	30 31	20 19			
001194	000	39 40	31 30			
001195	000	20 21	10 9			

001196	000	31	32	21	20
001197	000	40	41	32	31
001198	000	21	22	11	10
001199	000	32	33	22	21
001200	000	41	42	33	32
001201	000	43	44	36	35
001202	000	44	45	37	36
001203	000	45	46	38	37
001204	000	46	47	39	38
001205	000	47	48	40	39
001206	000	48	49	41	40
001207	000	49	50	42	41
001208	000	51	52	44	43
001209	000	52	53	45	44
001210	000	53	54	46	45
001211	000	54	55	47	46
001212	000	55	56	48	47
001213	000	56	57	49	48
001214	000	57	58	50	49
001215	000	59	60	52	51
001216	000	60	61	53	52
001217	000	61	62	54	53
001218	000	62	63	55	54
001219	000	63	64	56	55
001220	000	64	65	57	56
001221	000	65	66	58	57
001222	000	67	68	60	59
001223	000	68	69	61	60
001224	000	69	70	62	61
001225	000	70	71	63	62
001226	000	71	72	64	63
001227	000	72	73	65	64
001228	000	73	74	66	65
001229	000	75	76	68	67
001230	000	76	77	69	68
001231	000	77	78	70	69
001232	000	78	79	71	70
001233	000	79	80	72	71
001234	000	80	81	73	72
001235	000	81	82	74	73
001236	000	83	84	76	75
001237	000	84	85	77	76
001238	000	85	86	78	77
001239	000	86	87	79	78
001240	000	87	88	80	79
001241	000	88	89	81	80
001242	000	89	90	82	81
001243	000	2	9	4	10
001244	000	2	50.		
001245	000	1	1	2	
001246	000	2	2	3	

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APPENDIX 2

DATA INPUT FORMAT

Card 1: FORMAT (20A4)

TITLE - Title of the problem

Card 2: FORMAT (10I5)

(1) INODE - No. of nodes

(2) NELEM - No. of elements

(3) NAPC - No. of applied point load

(4) NBC - No. of interface element

(5) NINCR - No. of load increment

(6) NCYCL - a dummy

(7) ICASE = 0 for plastic analysis

(8) NRIGD - No. of rigid element

(9) NULOAD - No. of uniformly loaded element

Card 3: FORMAT (8F10.4)

(1) YSTRS - a dummy

(2) DELL - "

(3) ZETA - "

(4) PMAX - Maximum load one wants to apply

(5) DLMAX - a dummy

Card 4: FORMAT (4F20.5)

(1) DZI - Shear modulus for interface element

(2) DRI - Rotational modulus for interface

Card 5: FORMAT (4F20.5)

- (1) EC - Modulus of elasticity for rigid element
- (2) XNUC - Poisson's ratio for rigid element
- (3) ES - Modulus of elasticity for soil
- (4) XNUS - Poisson's ratio for soil

Card 6: FORMAT (8F10.4)

- (1) PI - Angle of friction in degree
- (2) SMALLK - Swelling index
- (3) XI - Adhesion (for interface element)
- (4) VOIDI - Initial void ratio
- (5) PO - Initial density
- (6) DEPTH - Maximum depth of soil
- (7) ALAMDA - Compression index
- (8) EPSLON - Angle of friction for interface element

Card 7: FORMAT (2F10.4, 2I5)

- (1) Z(I) - Z - Coordinate value (downward positive)
- (2) R(I) - R - Coordinate value
- (3) IZ = 0 if free to Z-direction
= 1 if note
- (4) IR = 0 if free to R-direction
= 1 if note

Repeat INODE times in the order of node number

Card 8: FORMAT (4I5)

4 node numbers of an element in counter-clockwise. Repeat NELEM times in the order of element number. Ordering of element should be:

- (1) Rigid element (2) interface element
- (3) soil element

* Card(s) 9: FORMAT (10I5)

- (1) ID(I,1) - Element number left to I^{th} interface element
- (2) ID(I,2) - Element number right to I^{th} interface element

Repeat NBC/5 times with 5 sets of data on one card

* Not required if NBC=0

Card(s) 10: FORMAT (I5, 2F15.6)

- (1) NODE - Node number with point load
- (2) PZ - Z-component
- (3) PR - R-component

Repeat NAPC times

Not required if NAPC=0

** Card 11: FORMAT (I5, F20.9)

- (1) NLDEL - No. of uniformly loaded elements
- (2) ULOAD - Load intensity (compression is positive)

** Card(s) 12: FORMAT (3I5)

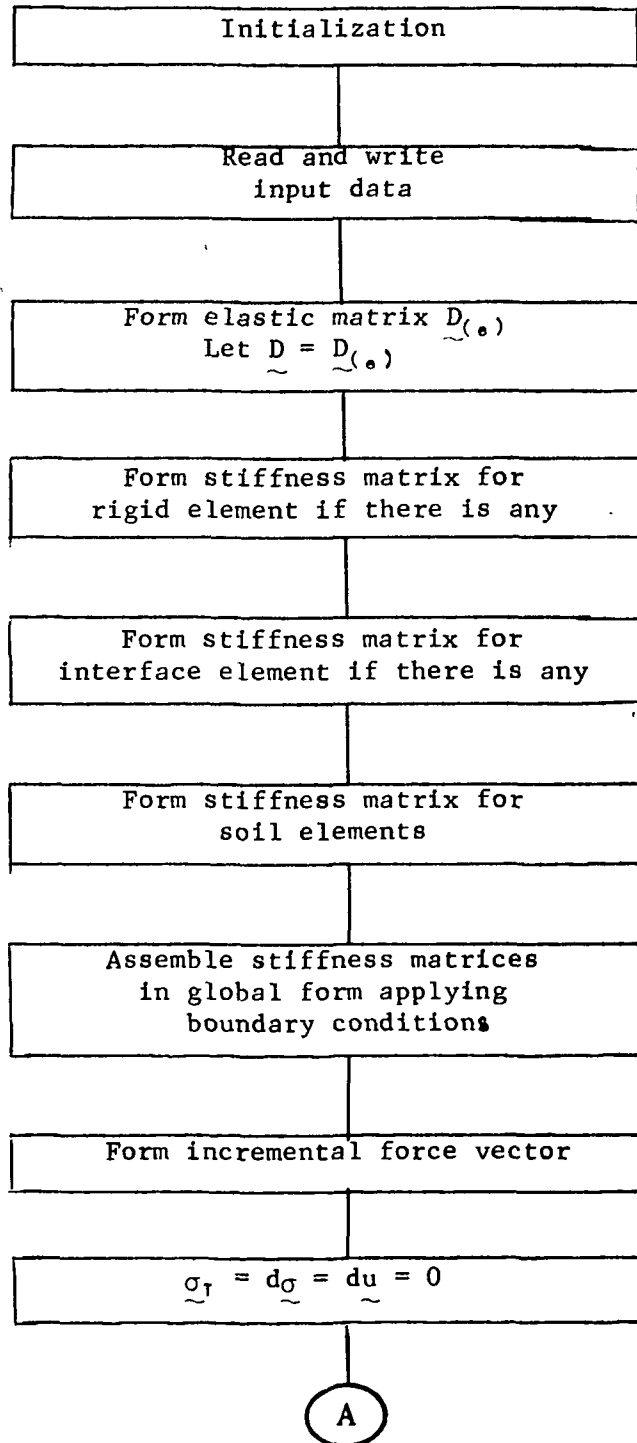
- (1) NOL - Loaded element number
- (2) NLFT - Node No. at left
- (3) NRHT - Node No. at right

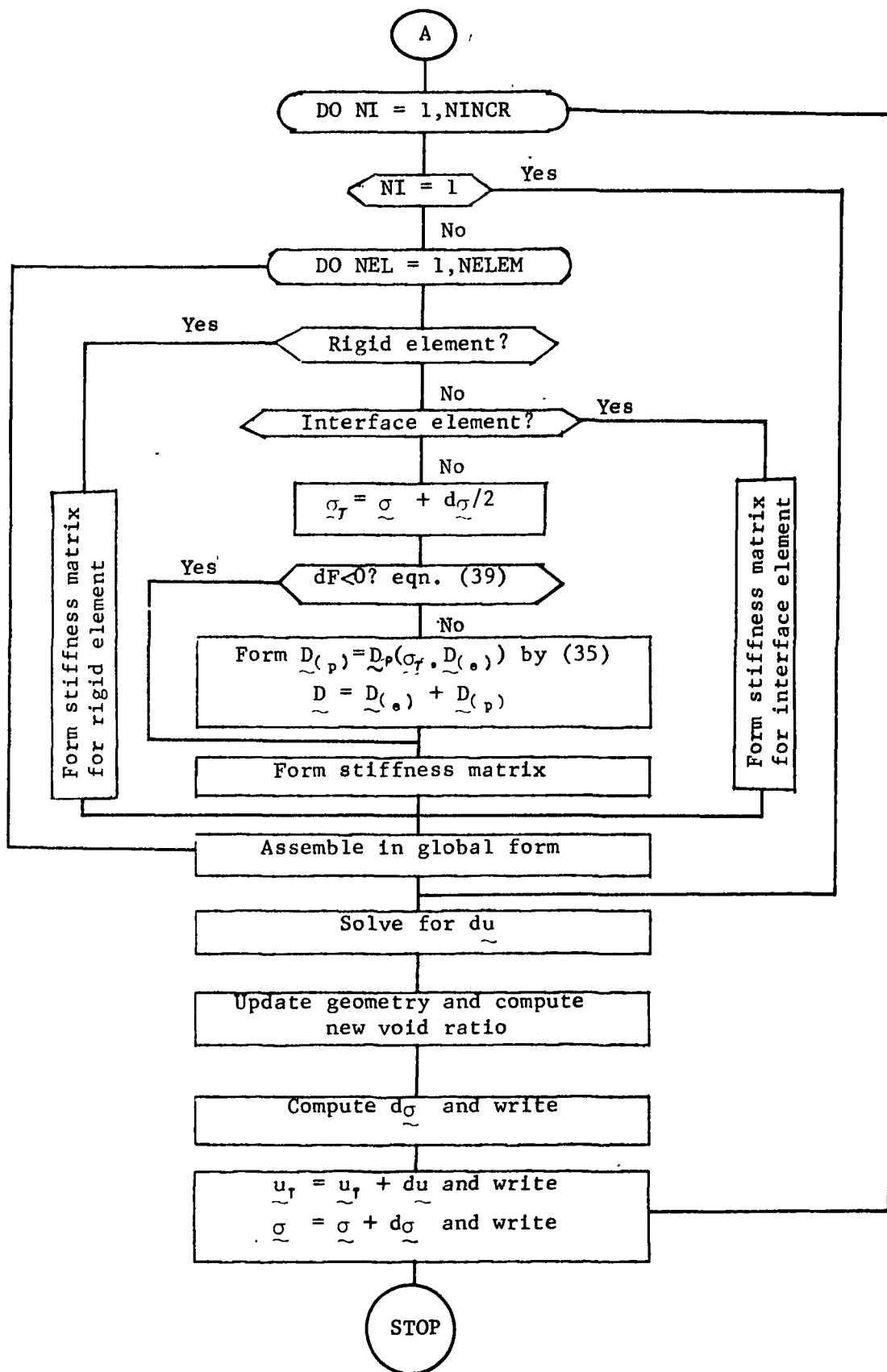
Repeat NLDEL times

** Not required if NULOAD=0

APPENDIX 3

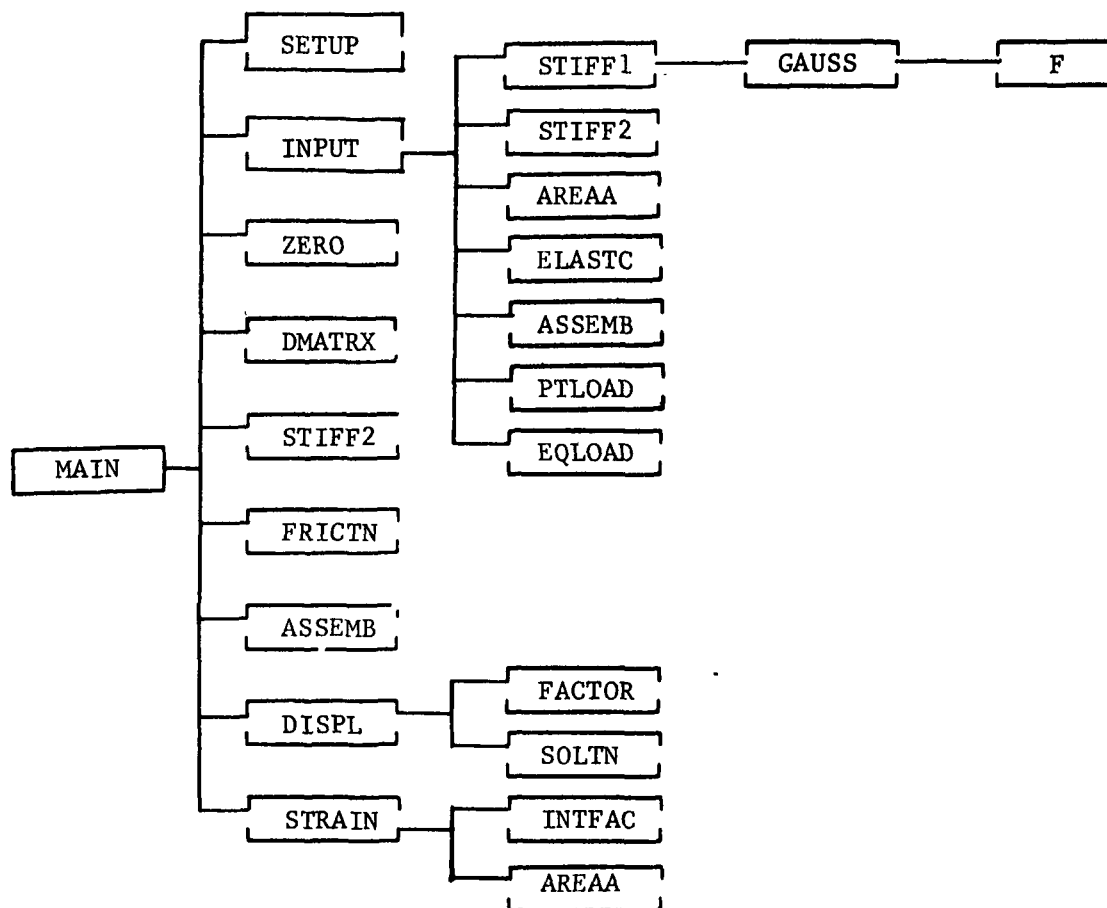
FLOW CHART





APPENDIX 4

SUBROUTINE ORGANIZATION CHART



APPENDIX 5

DESCRIPTIONS OF SUBROUTINES

Subroutine Name	DESCRIPTIONS
SETUP	Assigns necessary constants for integrations
INPUT	Reads and writes input data, and gets ready for the first linear elastic solution.
ZERO	Clear a given matrix.
DMATRX	Computes $D_{(p)}$ and forms $D = D_{(e)} + D_{(p)}$ if $dF \geq 0$ for a given element.
STIFF 1	Forms submatrices of stiffness matrices
GAUSS	Integrates by Gaussian quadrature
F	Gives functions to be integrated
FRICTN	Updates interface moduli and forms interface element stiffness.
AREAA	Computes cross sectional area of an element.
ELASTC	Forms elastic matrix $D_{(e)}$
ASSEMB	Assembles stiffness matrices into global stiffness matrix applying boundary conditions.
PTLOAD	Reads and writes applied point load if there is any.
EQLOAD	Reads and writes applied uniform load and computes equivalent nodal forces.
DISPL	Calls FACTOR and SOLTN, and writes u for first 10 nodes.
FACTOR	Factors the given simultaneous eqns. in one dimensional array.
SOLTN	Backward substitution is performed to give a set of solutions.
STRAIN	Computes incremental strains and stresses. Also

computes new void ratio.

INTFAC Computes incremental stresses for interface element
if there is any.

STIFF2 Forms stiffness matrix using \underline{D} .

PART II

DYNAMICS OF WHEEL-SOIL INTERACTION

II -1. INTRODUCTION

Deformations and stresses of soil media under a moving wheel are complex phenomena. A rate-dependent inelastic behavior associated with inertia effects must be considered. Somewhat simplified analyses have been reported by various authors. Earlier contributions to the wheel-soil interaction by Bekker were followed by Micklethwaite [2], Evans [3], Uffelman [4], and Bekker [5]. Rigorous experimental and theoretical studies on this subject have also been reported by Onaffeko and Reece [6], Wong and Reece [7,8]. Yong and Webb [9] and Schuring [10] studied energy dissipation in soil-wheel interaction from the viewpoint of viscoplasticity. Windisch and Yong [11] further examined the strain-rate phenomena and presented a method of computing soil displacements and strain rates from the experiment-based "marker position". In contrast to these studies, Perumpral, Liljedahl and Perloff [12] used the finite element method to calculate stresses and deformations due to a rigid wheel interaction. They used variable modulus of elasticity determined from the stress-strain curve of the triaxial tests but ignored the effects of inertia and rate-dependency.

Elsamny and Ghobarah [13] studied the stress field in the soil mass under the loading of a rigid cylindrical wheel on the verge of

spinning. However, the fact that the kinematic characteristics of the wheel and the velocity boundary conditions on the wheel-soil interface is ignored has been criticized by Wong [14]. More recently, Kloc [15] presented analytical formulations on mechanical interaction of a driven roller on soil slopes. In this study, a gravitating cohesive-frictional soil was considered with Kötter's quasi-static equilibrium equations applied to a plastic stress configuration (Mohr-Coulomb criteria) satisfying Shield's velocity conditions along the characteristic lines. Energy dissipation was not considered in this study.

In the present study we propose a rational approach in which the rate-dependent inelastic properties together with effects of inertia are adequately taken into account. Equilibrium conditions for wheel-soil interaction reported by Onaffeko and Reece [6] and Wong and Reece [7] are used to obtain radial and tangential stresses at the interface. Although the nonisothermal conditions may be considered without special difficulties in the framework of continuum mechanics and irreversible thermodynamic process, the present study is limited to an isothermal condition. The Mohr-Coulomb failure criterion appears to dominate most of the wheel-soil interaction studies. However, in view of the fact that the soil behaves as a strain-hardening material, in general, rather than perfectly plastic or rigid plastic material, we will overcome such deficiency by using the concept of critical state soil mechanics.

In what follows we make use of the internal state variable approaches of Coleman and Gurtin [16] and Perzyna and Wojno [17]

However, a basic difference from their approach is introduced in the present study such that the free energy functional containing inelastic behavior is not considered smooth for its entire domain of histories. Rather, we assume a form of discretized free energy as a function of elastic strains, plastic strains and internal or hidden variables of incremental quantity considered to be valid only for a small time interval or a fraction of loading increments. Here the hidden variables may represent a viscous or physicochemical behavior, properties other than what is commonly known as "elastic" and "plastic". Once the form of incremental free energy containing all nonlinear functions is prescribed for a small time interval, then the superposition of these nonlinear terms is permissible. Namely, the plastic material kernel may be calculated from the independent viscoelastic responses within this small time interval. Thus the histories can be carried over from one time increment to another until desired histories are completed. This will be accomplished by a suitable difference operator.

To represent inelastic behavior of soil we use the concept of critical state [18] and yield surface of Roscoe and Burland [19]. A derivation of the plastic tangent stiffness matrix based on this theory in the context of incremental theory of plasticity and its finite element applications were presented in Part I of this report. It should be noted that the particular internal state variable approach used here in conjunction with incremental free energy expression leads to a valid coupling of the completely independent plasticity theory and the rate dependent hidden variables.

Numerical examples are presented to demonstrate effectiveness of the present method. The well-known finite element method [20,21] is

utilized in the computation.

II - 2. BALANCE OF ENERGY AND LINEAR MOMENTUM

We record here the principle of conservation of energy which states that the time rate of change of the kinetic energy k plus the internal energy U is equal to R , the mechanical power on the system.

$$\dot{k} + \dot{U} = R \quad (1)$$

Here the superposed dot indicates a time rate, and

$$k = \frac{1}{2} \int_V \rho v_i v_i dV \quad (2)$$

$$U = \int_V \rho e dV \quad (3)$$

$$R = \int_V \rho F^j v_j dV + \int_A s^{ij} v_j n_i dA \quad (4a)$$

in which ρ is the density, v_i is the velocity component; e is the internal energy density; F^i is the body force; s^{ij} is the surface traction; and n_i is the unit normal to the surface. Using the Green - Gauss theorem, (4a) becomes

$$R = \int_V (\rho F^j v_j + \sigma^{ij} v_{j,i} + \sigma_{,i}^{ij} v_j) dV \quad (4b)$$

Now, inserting (2) and (4b) into (1) yields

$$\int_V [(\sigma_{,i}^{ij} + \rho F^j - \rho a^j) v_j - \rho \dot{e} + \sigma^{ij} v_{j,i}] dV = 0 \quad (5)$$

For the principle of balance of linear momentum to hold and for arbitrary volumes we must have

$$\sigma_{,i}^{ij} + \rho F^j - \rho a^j = 0 \quad (6)$$

and

$$\rho \dot{\epsilon} = \sigma^{ij} v_{j,i} = \sigma^{ij} \dot{\gamma}_{ij} \quad (7)$$

Here σ^{ij} and γ_{ij} are the stress tensor and strain tensor; the comma denotes ordinary differentiation; and a^j is the acceleration. It should be noted that equations (2) through (7) refer to rectangular cartesian coordinates. We regard (7) as the balance of energy.

II - 3. INCREMENTAL FREE ENERGY FUNCTIONS

In view of the earlier discussion our objective is to propose a form of free energy functions in incremental quantity such that the non-smooth or inelastic strains may be included for a small time interval Δt . For isothermal conditions, the incremental free energy $\phi(\Delta t)$ and stresses $\sigma^{ij}(\Delta t)$ are assumed to be functions of incremental strains $\gamma_{ij}(\Delta t) = \gamma_{ij}^{(e)}(\Delta t) + \gamma_{ij}^{(p)}(\Delta t)$ and incremental internal state variables (or hidden variables) $\alpha_{ij}^{(r)}(\Delta t) = \alpha_{ij}^{(r)(e)}(\Delta t) + \alpha_{ij}^{(r)(p)}(\Delta t)$ where (e) and (p) represent elastic and plastic components, respectively. This statement may be given by

$$\phi(\Delta t) = \hat{\phi}[\gamma_{ij}^{(e)}(\Delta t), \gamma_{ij}^{(p)}(\Delta t), \alpha_{ij}^{(r)(e)}(\Delta t), \alpha_{ij}^{(r)(p)}(\Delta t)] \quad (8)$$

$$\sigma^{ij}(\Delta t) = \hat{\sigma}[\gamma_{ij}^{(e)}(\Delta t), \gamma_{ij}^{(p)}(\Delta t), \alpha_{ij}^{(r)(e)}(\Delta t), \alpha_{ij}^{(r)(p)}(\Delta t)] \quad (9)$$

For isothermal conditions, the free energy is the same as the internal energy so that

$$\rho \dot{\phi} = \rho \dot{\epsilon} = \sigma^{ij} \dot{\gamma}_{ij}$$

or for the small time interval Δt ,

$$\sigma^{ij}(\Delta t) = \sigma^{ij}(\Delta t) (\dot{\gamma}_{ij}^{(e)}(\Delta t) + \dot{\gamma}_{ij}^{(p)}(\Delta t)) \quad (10)$$

At this point we introduce here the incremental form of free energy in a truncated Taylor series expansion,

$$\begin{aligned} \rho \dot{\phi}(\Delta t) = & \frac{1}{2} E^{ijkl} \gamma_{ij}^{(e)} \gamma_{kl}^{(e)} + \frac{1}{2} E^{*ijkl} \gamma_{ij}^{(p)} \gamma_{kl}^{(p)} \\ & + \frac{1}{2} \sum_{r=1}^n E_{(r)}^{ijkl} (\alpha_{ij}^{(e)} + \alpha_{ij}^{(p)}) (\alpha_{kl}^{(e)} + \alpha_{kl}^{(p)}) \\ & + \sum_{r=1}^n E_{(r)}^{ijkl} (\alpha_{kl}^{(e)} + \alpha_{kl}^{(p)}) (\gamma_{ij}^{(e)} + \gamma_{ij}^{(p)}) \end{aligned} \quad (11)$$

where E^{ijkl} and E^{*ijkl} represent tensors of elastic and plastic moduli, respectively; $E_{(r)}^{ijkl}$ are stiffness constants associated with the internal variables. Note that (11) has the form of truncated Taylor series expansion only to include quadratic terms. However, the product term of $\gamma_{ij}^{(e)}$ and $\gamma_{ij}^{(p)}$ is missing. This is because the coupling of elastic and plastic strains can be obtained using any one of the failure theories and an explicit material kernel relating the product of $\gamma_{ij}^{(e)}$ and $\gamma_{ij}^{(p)}$ is nonexistent.

Lastly, $\alpha_{ij}^{(r)}$ defined here as the internal variables represent time dependent physicochemical properties or simply a viscous behavior which may be expressed as

$$\alpha_{ij}^{(r)} = \int_0^t \exp \left[-\frac{(t-\tau)}{T_{(r)}} \right] \dot{\gamma}_{ij}(\tau) d\tau \quad (12)$$

where τ is the time variable and $T_{(r)}$ is the relaxation time. In order to facilitate an explicit integration we assume a linear variation of $\dot{\gamma}_{ij}$ within the time interval Δt given by

$$\dot{\gamma}_{ij}(s) = \dot{\gamma}_{ij}(s-1) + \frac{\tau - (t - \Delta t)}{\Delta t} (\dot{\gamma}_{ij}(s) - \dot{\gamma}_{ij}(s)) \quad (13)$$

where s is the current time step. Substituting (13) in (12) and performing integration we obtain

$$\begin{pmatrix} r \end{pmatrix}_{\alpha_{1j}}(s) = \begin{pmatrix} r \end{pmatrix} \begin{pmatrix} r \end{pmatrix}_{\alpha_{1j}}(s-1) + \begin{pmatrix} r \end{pmatrix} \dot{\gamma}_{1j}(s-1) + \begin{pmatrix} r \end{pmatrix} \dot{\gamma}_{1j}(s) \quad (14)$$

in which

$$\begin{pmatrix} r \end{pmatrix} A = \exp\left(\frac{-\Delta t}{T(r)}\right), \quad \begin{pmatrix} r \end{pmatrix} B = T(r) \left(\begin{pmatrix} r \end{pmatrix} D - \begin{pmatrix} r \end{pmatrix} A \right)$$

$$\begin{pmatrix} r \end{pmatrix} C = T(r) (1 - \begin{pmatrix} r \end{pmatrix} D), \quad \begin{pmatrix} r \end{pmatrix} D = \frac{T(r)}{\Delta t} (1 - \begin{pmatrix} r \end{pmatrix} A)$$

The derivation of these parameters is given in Appendix 1.

Rewriting (10) for the current time step (s) as

$$\rho \left\{ \frac{\partial \Phi(s)}{\partial \gamma_{1j}(s)} \dot{\gamma}_{1j}^{(e)}(s) + \frac{\partial \Phi(s)}{\partial \gamma_{1j}^{(p)}(s)} \dot{\gamma}_{1j}^{(p)}(s) + \frac{\partial \Phi(s)}{\partial \alpha_{1j}(s)} \begin{pmatrix} r \end{pmatrix}_{\alpha_{1j}}(s) + \frac{\partial \Phi(s)}{\partial \alpha_{1j}(s)} \begin{pmatrix} r \end{pmatrix}_{\alpha_{1j}}^{(p)}(s) \right\} - \sigma^{1j}(s) (\dot{\gamma}_{1j}^{(e)}(s) + \dot{\gamma}_{1j}^{(p)}(s)) = 0 \quad (15)$$

and substituting (14) and (11) into (15) yields

$$\begin{aligned} & \{ E^{1jkl} \gamma_{kl}^{(e)}(s) + \sum_{r=1}^n E_{(r)}^{1jkl} \left(\begin{pmatrix} r \end{pmatrix} \begin{pmatrix} r \end{pmatrix}_{\alpha_{kl}}(s-1) + \begin{pmatrix} r \end{pmatrix} \dot{\gamma}_{kl}^{(e)}(s-1) + \begin{pmatrix} r \end{pmatrix} \dot{\gamma}_{kl}^{(e)}(s) \right. \\ & \quad \left. - \sigma^{1j}(s) \right\} \dot{\gamma}_{1j}^{(e)}(s) + \sum_{r=1}^n E_{(r)}^{1jkl} \left(\begin{pmatrix} r \end{pmatrix}_{\alpha_{1j}}(s) \begin{pmatrix} r \end{pmatrix}_{\alpha_{kl}}^{(p)}(s) + \begin{pmatrix} r \end{pmatrix}_{\alpha_{kl}}^{(p)}(s) \dot{\gamma}_{1j}^{(e)}(s) \right. \\ & \quad \left. + \begin{pmatrix} r \end{pmatrix}_{\alpha_{kl}}^{(p)}(s) \dot{\gamma}_{1j}^{(p)}(s) + \gamma_{1j}^{(r)} \alpha_{kl}^{(r)} - \sigma^{1j}(s) \dot{\gamma}_{1j}^{(p)}(s) \right) = 0 \end{aligned}$$

Since all variations other than $\dot{\gamma}_{1j}^{(e)}$ are not arbitrary we must have

the relationship

$$\sigma^{1j}(s) = E^{1jkl} \gamma_{kl}^{(e)}(s) + \sum_{r=1}^n E_{(r)}^{1jkl} \left(\begin{pmatrix} r \end{pmatrix} \begin{pmatrix} r \end{pmatrix}_{\alpha_{kl}}(s-1) + \begin{pmatrix} r \end{pmatrix} \dot{\gamma}_{kl}^{(e)}(s-1) + \begin{pmatrix} r \end{pmatrix} \dot{\gamma}_{kl}^{(e)}(s) \right) \quad (16)$$

$$\begin{aligned}
& \frac{1}{E} \epsilon_{ijk} \dot{\gamma}_{ij}^{(p)}(s) - \sigma_{ij}^{(s)} \dot{\gamma}_{ij}^{(p)} + \sum_{r=1}^n \epsilon_{ijk} \left[\alpha_{ij}^{(r)}(s) \dot{\alpha}_{kl}^{(r)}(p)(s) \right. \\
& \left. + \alpha_{kl}^{(r)}(s) \dot{\gamma}_{ij}^{(p)}(s) + \alpha_{kl}^{(r)}(s) \dot{\gamma}_{ij}^{(s)} + \gamma_{ij}^{(r)} \dot{\alpha}_{kl}^{(r)}(s) \right] = 0
\end{aligned} \quad (17)$$

Here (16) represents the relationship

$$\sigma_{ij} = \rho \frac{\partial \phi}{\partial \gamma_{ij}^{(s)}}$$

which states that the stresses are derivable from the free energy functions. It should be noted that, in our specific problem, this stress is due to an elastic strain and a law governing the plastic strain and stress is needed to obtain the stress due to a total strain. The relationship (17) may be considered as the dissipation which plays a significant role in heat conduction problems. However, for the isothermal conditions as considered in the present study, the entire terms of (17) need not be used in the analysis. Only the first term will be recovered as we apply a yield criterion in (16).

II - 4. INELASTIC RESPONSE

Extensive research has been carried out at Cambridge University by Roscoe and his colleagues [19] on the subject of the critical state soil mechanics. The yield criteria adopted here were originally proposed by Roscoe and Burland [19]. A plastic tangent matrix in the context of the incremental theory of plasticity was derived by the authors [22, 23, 24]. A new method of checking conditions of yielding is elaborated in Part I of this report. For the purpose of reference we repeat the expression for the incremental stress associated with rate-independent

elastoplastic behavior,

$$d\sigma^{ij} = (E^{ijkl} + \overset{*}{E}^{ijkl}) d\gamma_{kl} \quad (18)$$

A close examination of (16) reveals that $\sigma^{ij(s)}$ is the total stress due to the elastic component of strain and internal variable for the current time step. On the other hand, (18) represents an incremental stress for a fraction of loading increments with inelastic strain coupled. It is then immediately clear that if the viscoelastic stress as given by (16) is used to calculate $\overset{*}{E}^{ijkl}$ within the time interval and if we proceed with (18) with iterative cycling for further updating $\overset{*}{E}^{ijkl}$ without participation of the viscous part of (16), then at the end of the time interval the total strain reached simply reflects the coupling of viscoelastic and plastic properties. Thus from (16) and (20), we obtain,

$$\begin{aligned} d\sigma^{ij(s)} = & E^{ijkl} d\gamma_{kl}^{(s)} + \sum_{r=1}^n \overset{*}{E}^{ijkl(r)} \left(\overset{(r)}{A} d\alpha_{kl}^{(s-1)} \right. \\ & \left. + \overset{(r)}{B} d\dot{\gamma}_{kl}^{(s-1)} + \overset{(r)}{C} d\dot{\gamma}_{kl}^{(s)} \right) + \overset{*}{E}^{ijkl} d\dot{\gamma}_{kl}^{(s)} \end{aligned} \quad (19)$$

Note that viscoelastic strain is now associated with the total strain as coupling is established.

II - 5. FINITE ELEMENT EQUATIONS OF MOTION

The finite element method is widespread in engineering applications [11,12]. No elaboration on this method is attempted here.

In view of (7), (2) and (3) we rewrite (1) as

$$\int_V \rho \ddot{u}_i \dot{u}_i dV + \int_V \sigma^{ij} \dot{\gamma}_{ij} dV - \int_V \hat{F}^m \dot{u}_m dV = 0 \quad (20)$$

Here the body force $\rho F^m = \hat{F}^m$ alone is considered merely for simplicity. The surface traction can easily be added later if needed.

In the present study we use the plane strain isoparametric element with 4 corner nodes. This gives the linear variation of displacements in the form

$$u_i = \psi_N u_i^N \quad (21)$$

where ψ_N is the interpolation function and u_i^N is the nodal values of displacements u_i ($i = 1, 2$) and $N = 1, 2, 3, 4$.

The strain tensor is given by

$$\gamma_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}) \quad (22)$$

Inserting (25) in (26) yields

$$\gamma_{ij} = A_{Nij}^k u_k^N \quad (23)$$

where

$$A_{Nij}^k = \frac{1}{2}(\psi_{N,i} \delta_j^k + \psi_{N,j} \delta_i^k) \quad (24)$$

In view of (21), (23) and (20) we have

$$\left\{ \int_V \rho \psi_N \ddot{u}_k^N dV + \int_V \sigma^{ij} A_{Nij}^k dV - \int_V \hat{F}^k \psi_N dV \right\} \dot{u}_k^N = 0 \quad (25)$$

For all arbitrary values of \dot{u}_k^N we require the terms inside the bracket to vanish, which yields

$$M_{MN} \ddot{u}_k^N + \int_V \sigma^{ij} A_{Nij}^k dV = F_{Nk} \quad (26)$$

where M_{MN} and F_{Nk} are the mass matrix and the force vector, respectively,

$$M_{MN} = \int_V \rho \psi_M \psi_N dV \quad (27)$$

$$F_{Nk} = \int_V F^k \psi_N dV \quad (28)$$

To obtain an incremental form of (26), we take a variation or induce a perturbation such that

$$M_{MN} d\dot{u}_k^M + \int_V d\sigma^{ij} A_{Nij}^k dV = dF_{Nk} \quad (29)$$

Introducing the incremental stress (19) into (29) yields

$$M_{MN} d\dot{u}_k^M(s) + C_{MN}^{\ell k} d\dot{u}_\ell^M(s) + \left(\overset{(e)}{K}_{MN}^{\ell k} + \overset{(p)}{K}_{MN}^{\ell k} \right) du_\ell^M(s) = dF_{Nk}(s) + dF_{Nk}^{(v)}(s) \quad (30)$$

in which $C_{MN}^{\ell k}$, $\overset{(e)}{K}_{MN}^{\ell k}$, and $\overset{(p)}{K}_{MN}^{\ell k}$ are the viscosity matrix, elastic stiffness matrix and plastic stiffness matrix, respectively,

$$C_{MN}^{\ell k} = \int_V \sum_{r=1}^n \xi_{(r)}^{ijn(r)} \ell C A_{Mij}^k A_{Nmn}^k dV \quad (31)$$

$$\overset{(e)}{K}_{MN}^{\ell k} = \int_V E^{ijn} \ell A_{Mij}^k A_{Nmn}^k dV \quad (32)$$

$$\overset{(p)}{K}_{MN}^{\ell k} = \int_V E^{ijn} \ell A_{Mij}^k A_{Nmn}^k dV \quad (33)$$

The pseudo viscous load vector $dF_{Nk}^{(v)}$ is given by

$$\begin{aligned} dF_{Nk}^{(v)} &= \int_V \sum_{r=1}^n \xi_{(r)}^{ijn(r)} A_{Nmn}^k \alpha_{mn}^{(s-1)} A_{Mij}^k dV \\ &+ \int_V \sum_{r=1}^n \xi_{(r)}^{ijn(r)} \ell B A_{Mij}^k A_{Nmn}^k dV \{ d\dot{u}_\ell^M(s-1) \} \end{aligned} \quad (34)$$

The expression (30) is called the finite element equations of motion.

II - 6. SOLUTION PROCEDURE FOR INCREMENTAL EQUATIONS OF MOTION

A solution of (30) can easily be obtained by any scheme of direct numerical integration [13]. In this study, a constant acceleration for a small time increment is assumed, which gives a recurrence formula for displacements, velocities and accelerations in the form,

$$\{M_{MN} + \frac{\Delta t}{2} C_{MN} + \frac{\Delta t^2}{4} (\overset{(s)}{K}_{MN}^{\ell_k} + K_{MN}^{\ell_k})\} \ddot{u}_\ell^M(s) = dF_{Nk}^{(s)} - dF_{Nk}^{(v)}(s) - Q_{Nk}^{(s)} \quad (35)$$

where

$$Q_{Nk}^{(s)} = C_{MN} \left\{ \dot{u}_k^M(s-1) + \frac{\Delta t^2}{2} \ddot{u}_k^M(s-1) \right\} + (\overset{(s)}{K}_{MN}^{\ell_k} + \overset{(p)}{K}_{MN}^{\ell_k}) \left\{ u_\ell^M(s-1) + \frac{\Delta t^2}{4} \ddot{u}_\ell^M(s-1) \right\} \quad (36)$$

$$\dot{u}_\ell^M(s) = \dot{u}_\ell^M(s-1) + \frac{\Delta t}{2} \ddot{u}_\ell^M(s-1) + \frac{\Delta t}{2} \ddot{u}_\ell^M(s) \quad (37)$$

$$u_\ell^M(s) = u_\ell^M(s-1) + \frac{\Delta t^2}{4} \ddot{u}_\ell^M(s-1) + \frac{\Delta t^2}{4} \ddot{u}_\ell^M(s) + \Delta t \dot{u}_\ell^M(s-1) \quad (38)$$

Initially all terms associated with $(s-1)$ are zero and $\ddot{u}_\ell^M(s)$ in (35) can be solved from given initial and boundary conditions. Subsequently, $\dot{u}_\ell^M(s)$ and $u_\ell^M(s)$ are calculated from (38). These responses or histories are then carried to the next time increment and back to (35). However, for the second increment it is necessary to check yield con-

ditions and a standard incremental loading method of iteration [14] can be applied to each time increment with the total dynamic load on the structure.

II - 7. EQUIVALENT DYNAMIC WHEEL LOADS

Theoretical and experimental studies for the prediction of rigid and flexible wheel performance on soil have been reported by various authors as mentioned in Introduction. Onaffeko and Reece [6] presented practical procedures in determining radial and tangential stresses along the wheel-soil interface. Wong and Reece [7,8] derived expressions for sinkage, drawbar pull and torque input based on the plate penetration test but with considerations of the important aspects of the slip and the actual interaction between the wheels and soil.

In the present study the finite element equivalent nodal dynamic loadings are determined from the expressions for radial and tangential stresses given by Onaffeko and Reece [6] and explicit forms of these stresses as elaborated by Wong and Reece may also be used (See Appendix 2).

In order to compare the dynamic rate-dependent elastoplastic responses with the results of Perumpral, et al [12] who neglected the effects of inertia and rate-dependency, we consider here the identical geometry and material constants. The discretized wheel-soil medium is shown in Figure 1.

$E_s = 2,000 \text{ psi}$, $\nu_s = 0.45$
 $\phi = 36^\circ$, $e = 0.75$, $\nu = 0.05787 \text{ pci}$
 $T(r) = 0.1 \text{ sec. (r = 1,2,3)}$
 $\lambda = 0.05$, $\nu = 0.0001$

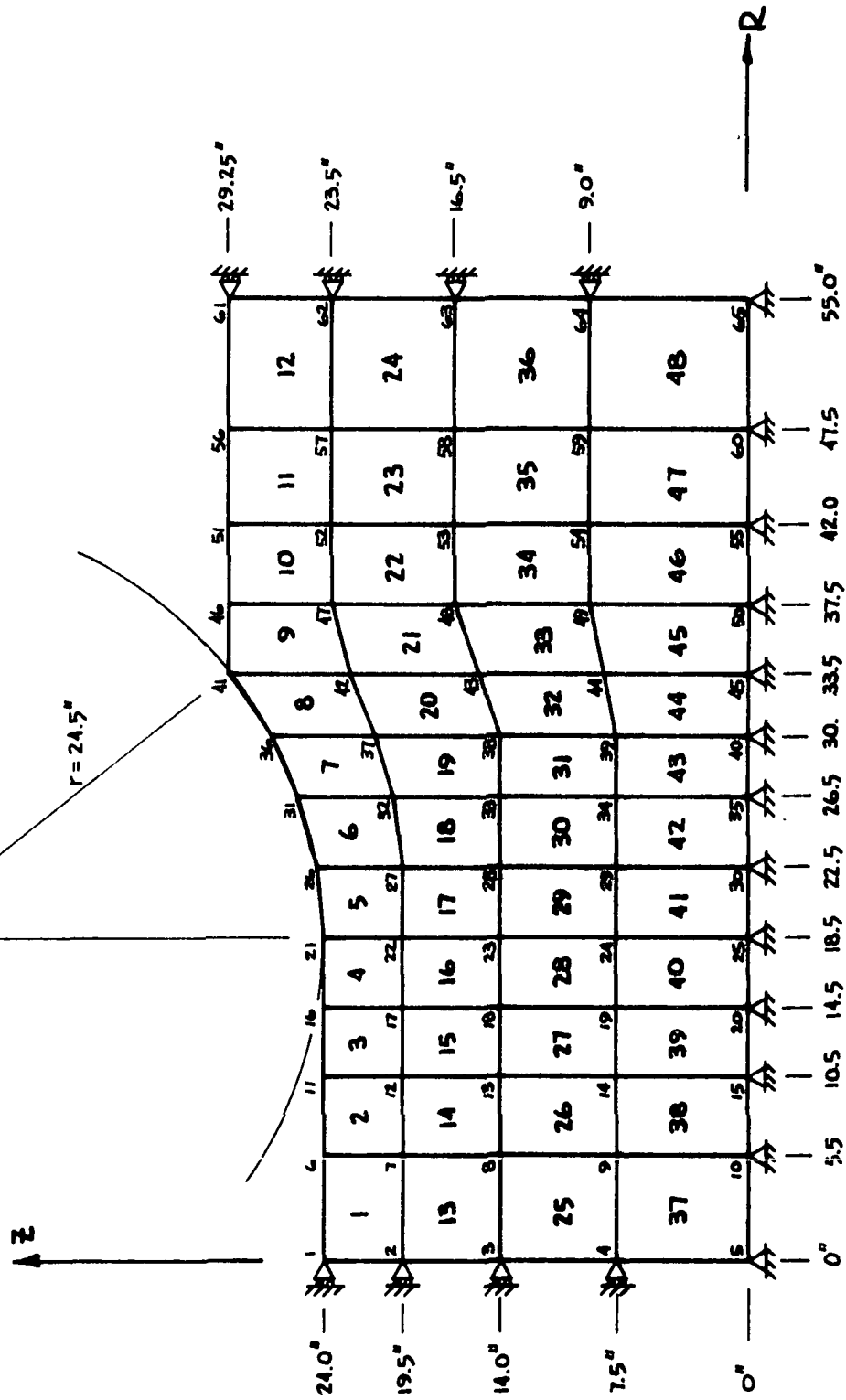


Figure 1: Wheel-Soil Interaction Geometry

Distributions of radial and tangential stresses on the rim of a 49 in. x 6 in. wide wheel on compact sand with 3.1% slip and 41.4% slip are shown in Figure 2 and Figure 3, respectively. Equivalent nodal loadings as calculated from the tributary area method in Figures 2 and 3 are shown in Figure 4. It is seen that the area under the curve corresponding to the wheel-soil contact area for each finite element may be conveniently approximated by the equivalent rectangular block. It should be noted that as the slippage increases the vertical downward loads decrease whereas the horizontal loads increase in the direction opposite to the wheel movement.

II - 8. DEFORMATION AND STRESS FIELDS

The equations of motion in assembled form for all finite elements are solved as described in Section II - 6. In order to compare the results for all possible effects, the computer program (Appendix 3) was written with many optional versions. Various cases studied include static analyses for elastic and elastoplastic responses and dynamic analyses for elastic, viscoelastic and viscoelastoplastic responses.

The material constants used are soil modulus $E_s = 2000$ psi, Poisson's ratio $\nu_s = 0.45$, angle of internal friction $\phi = 36^\circ$, density $\gamma = 0.05787$ pci, relaxation time $T_{(r)} = 0.1$ sec ($r = 1, 2, 3$), compression index $\lambda = 0.05$, and swelling index $\kappa = 0.0001$. These constants are chosen

to correspond to the compact sand which is used in the equivalent load representation as shown in Figures 2, 3, and 4. For dynamic analyses, a time increment $\Delta t = 0.0006$ Sec. for viscoelastoplastic response and $\Delta t = 0.0003$ Sec. for other responses are used.

Figure 5 shows these various responses at node No. 31. For static analyses, the elastoplastic displacement in the vertical direction is slightly larger than the elastic behavior. For dynamic analyses, the viscoelastic and viscoelastoplastic responses are considerably smaller than elastic and elastoplastic behavior. Once again, effects of plasticity result in larger deformations for both viscous and nonviscous cases.

The vector representations of elastoplastic deformations for the static analysis are shown in Figures 6 and 7. Deformations for 41.4% slip are larger than these for 3.1% slip. For the case of dynamic analysis (41.4% slip) the curvilinear transient deformation vectors for viscoelastoplastic response are shown in Figure 8. These vectors represent the time history from $t = 0$ to $t = 0.6$ sec. No doubt that the effects of inertia under dynamic loads caused larger deformations than under static loads but energy dissipation through the viscous behavior retarded the motion considerably in comparison with the nonviscous cases as noted in Figure 5. Deformed shapes for dynamic viscoelastoplastic responses at $t=0.3$ sec. are shown in Figure 9.

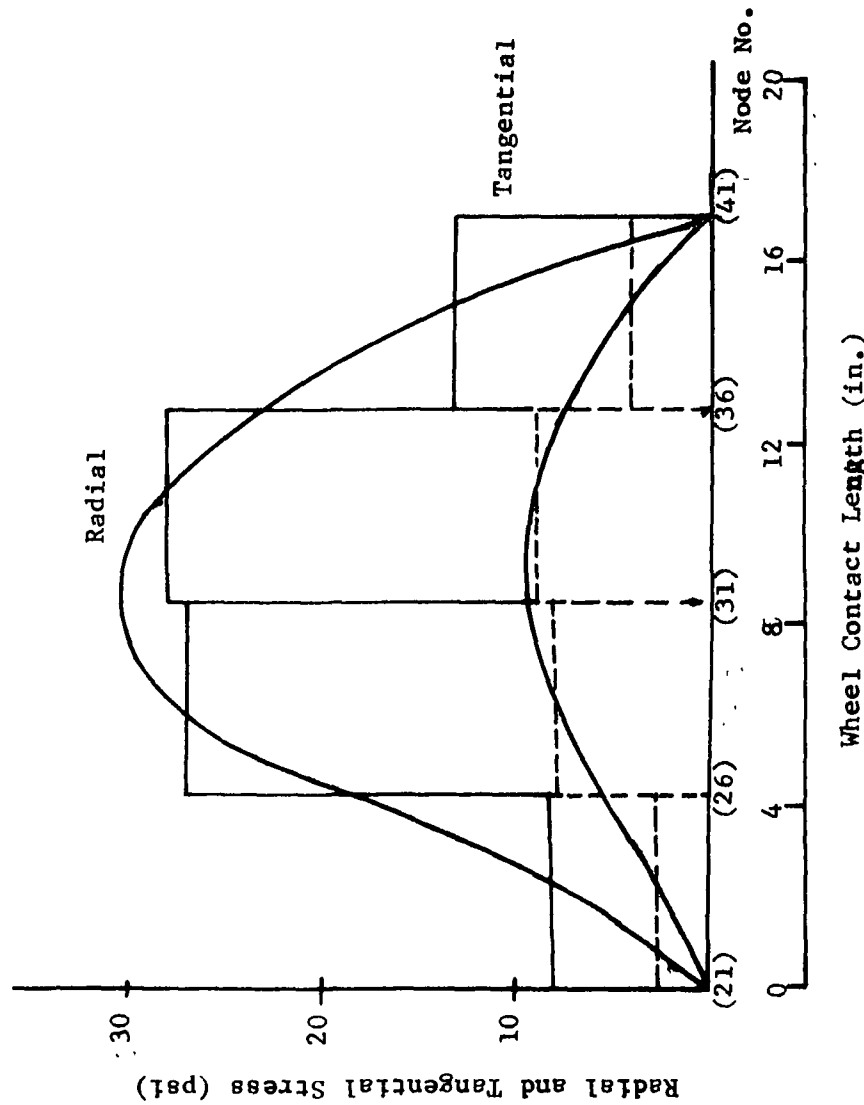


Figure 2:- Radial and Tangential Stress Distribution at the Interface for 3.1% Slip on Compact Sand. Ref. [6]

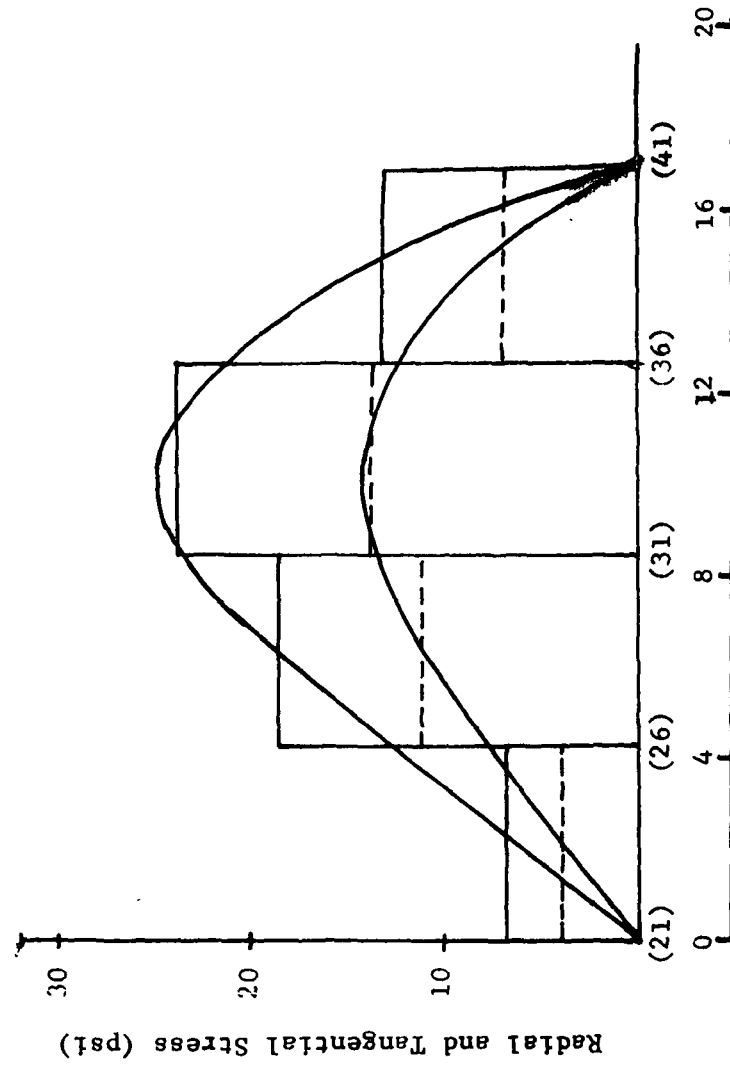
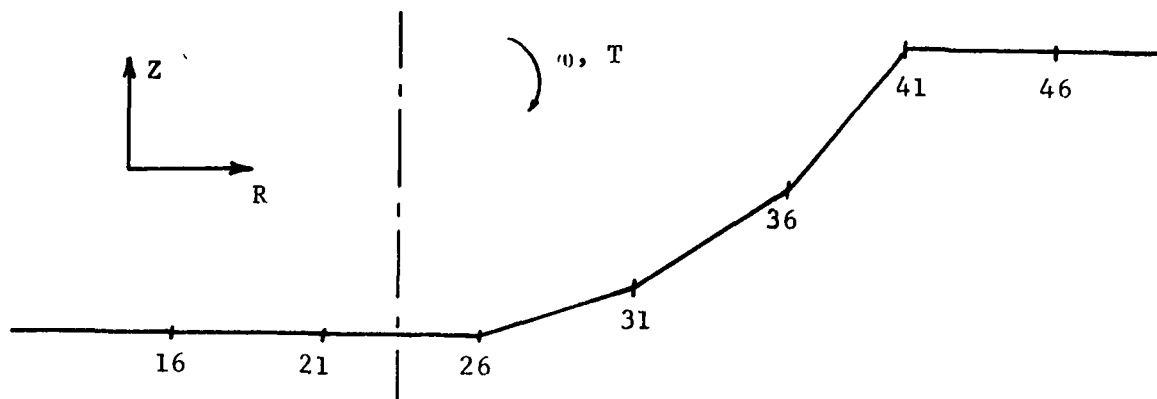


Figure 3: Radial and Tangential Stress Distribution at the Interface for 41.4% Slip on Compact Sand. Ref. [6]



NODE NO.	3.1% SLIP		41.4% SLIP	
	F_z (#)	F_R (#)	F_z (#)	F_R (#)
21	-16.17	-6.61	-13.42	-9.5
26	-74.5	-14.99	-55.21	-26.67
31	-119.62	-4.33	-98.27	-27.78
36	-88.86	-10.97	-89.1	-8.4
41	-27.11	8.3	-32.1	2.85

Figure 4: Equivalent Nodal Forces as determined from Figures 3 and 4.

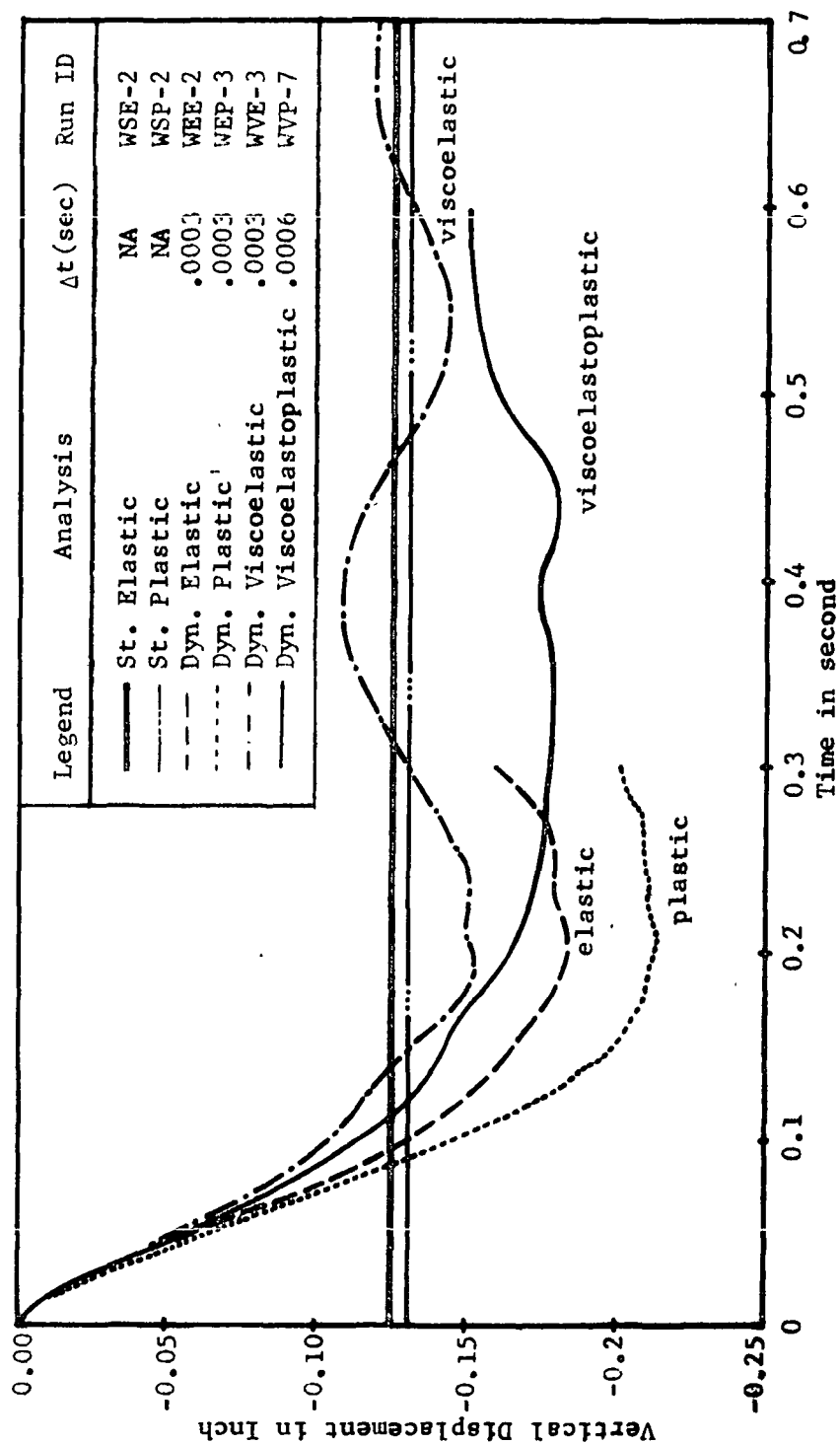


Figure 5: Time-Displacement Curves for 3.1% slip at Node No. 31.

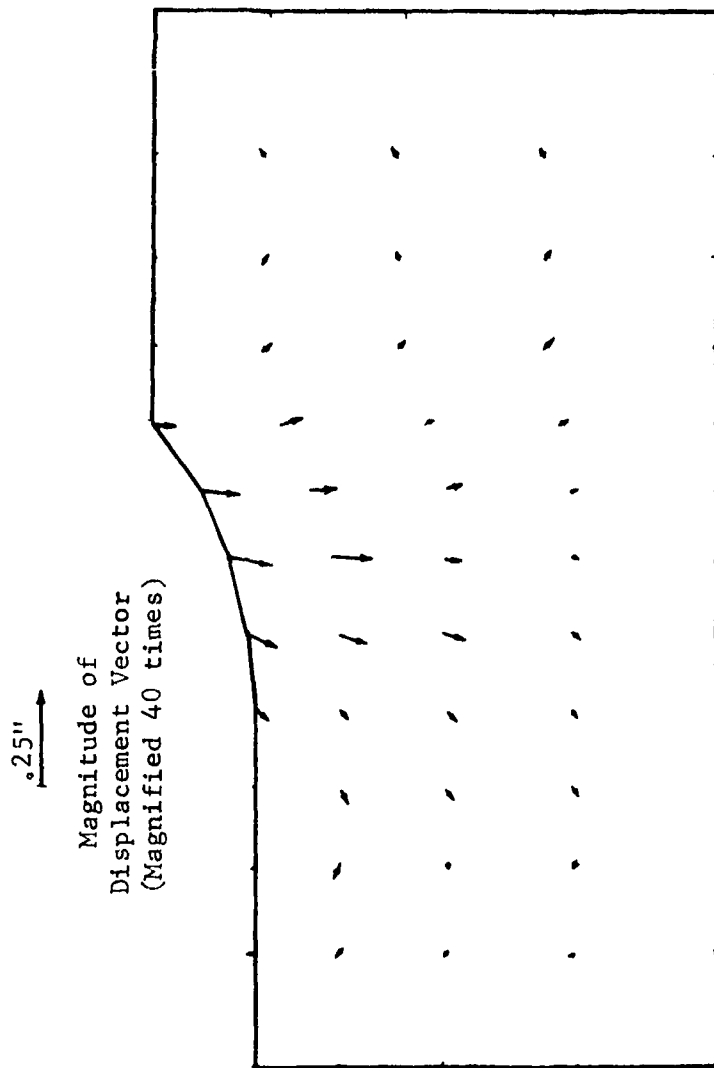


Figure 6: Vector Representation of Displacements (Static Elastoplastic Analysis for 3.1% Slip)

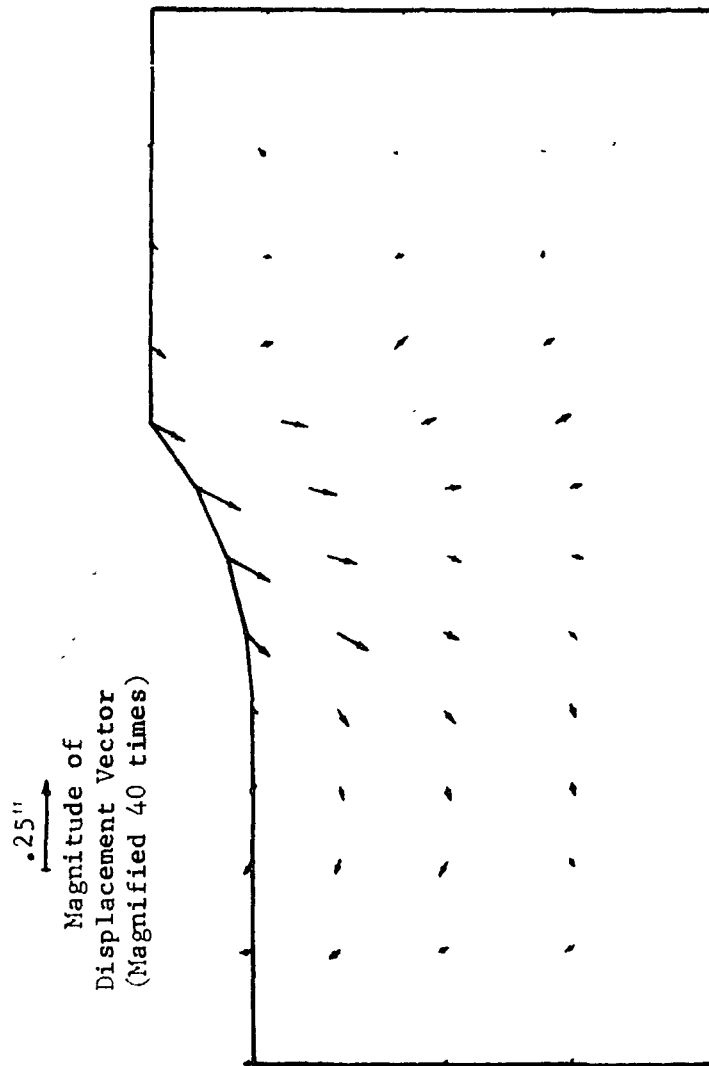


Figure 7: Vector Representation of Displacements (Static Elastoplastic Analysis for 41.4% Slip)

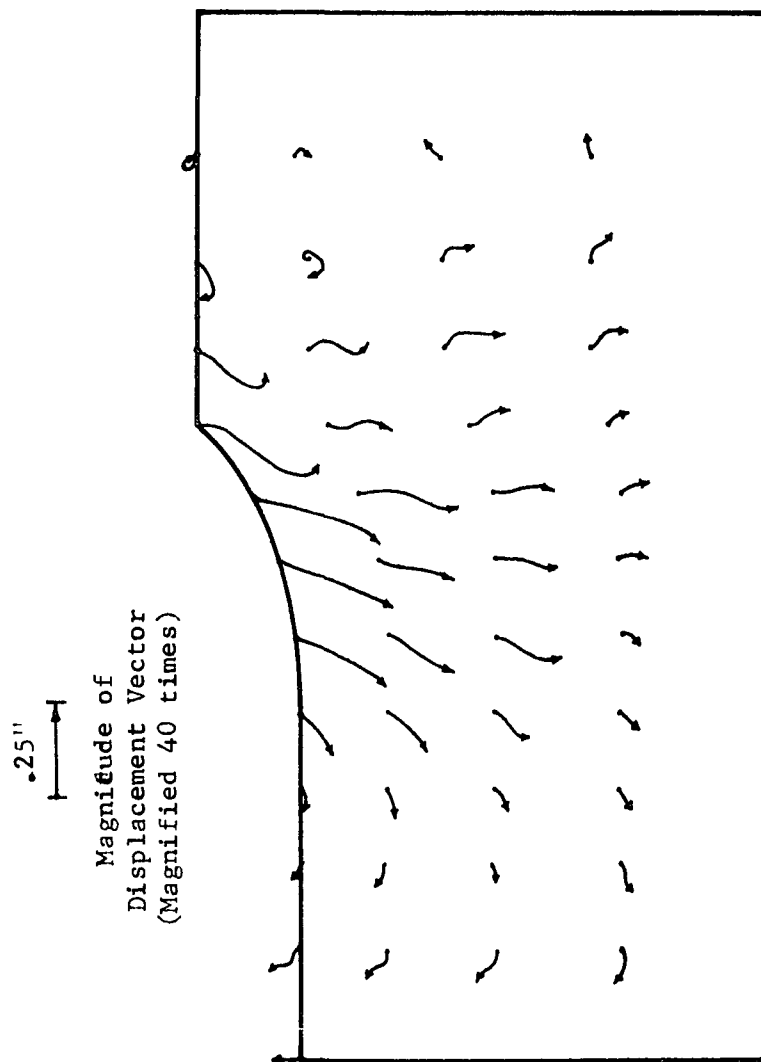


Figure 8: Vector Representation of Deformation (Dynamic Visco-elastoplastic Analysis for 41.4% Slip)

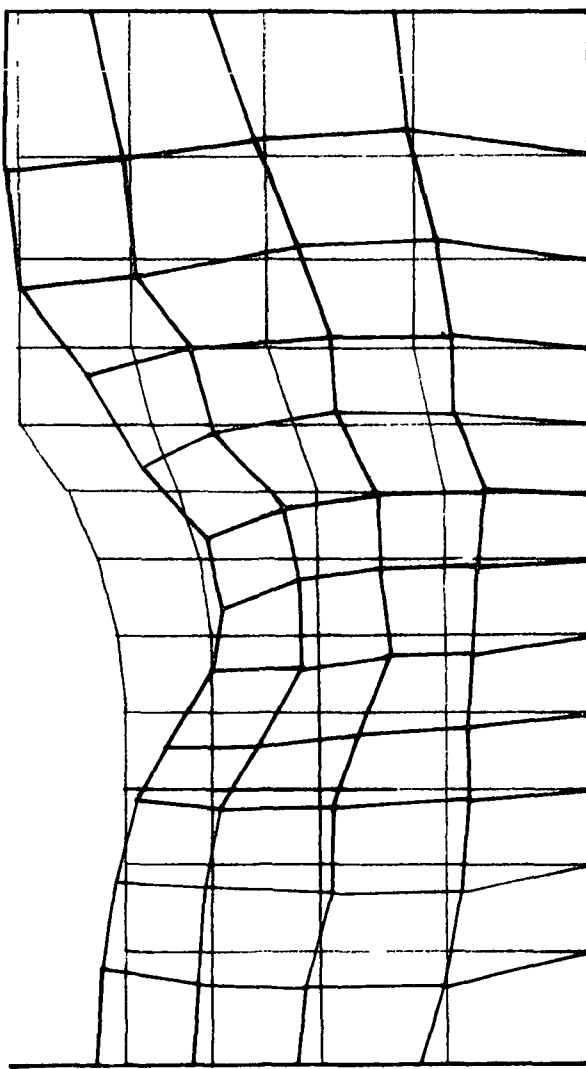


Figure 9: Deformed Configuration at .3 sec,
(Dynamic Viscoplastic Analysis for 41.4% Slip)

From the deformation fields various stress components are calculated and the results shown in the form of isobars in Figures 10 through 21. In the region close to the wheel the major principal stresses due to the elastoplastic deformations are smaller than those of Ref. [12] as shown in Figures 10 and 11 for 3.1% slip and 41.4% slip, respectively. Slightly larger major principal stresses develop at the mid-depth for the 3.1% slip. For the case of maximum shear stresses (Figures 12 and 13) the present analysis gives larger values than Ref. [12] for 3.1% slip, but this trend is reversed for 41.4% slip. In general, the maximum shear stresses for the 3.1% slip are larger than for the 41.4% slip, the same trend as in the case of major principal stresses. Dynamic elastoplastic major principal stresses and maximum shear stresses for 3.1% slip at $t=0.072$ sec., 0.15 sec., 0.228 sec., 0.3 sec., 0.6 sec. are shown in Figures 14 through 17. Variations of stresses with time until maximum stresses are reached are clearly shown. The effects of viscosity or rate-dependent plasticity for 3.1% slip at $t=0.3$ sec. and $t=0.6$ sec. are shown in Figures 18 and 19, respectively. The same information for 41.4% slip is given in Figures 20 and 21. It is seen that as the slip increases the major principal and maximum shear stresses tend to decrease.

II-9. CHARACTERIZATION OF SOIL MECHANICS PARAMETERS

Studies on deformation and stress fields as described in Section II-7 indicate that constitutive relationships for the soil behavior significantly influence the response patterns. The mechanics of wheel-soil

--- Reference 12
 --- Static Elastoplastic Analysis
 (Run ID. MSP - 2)

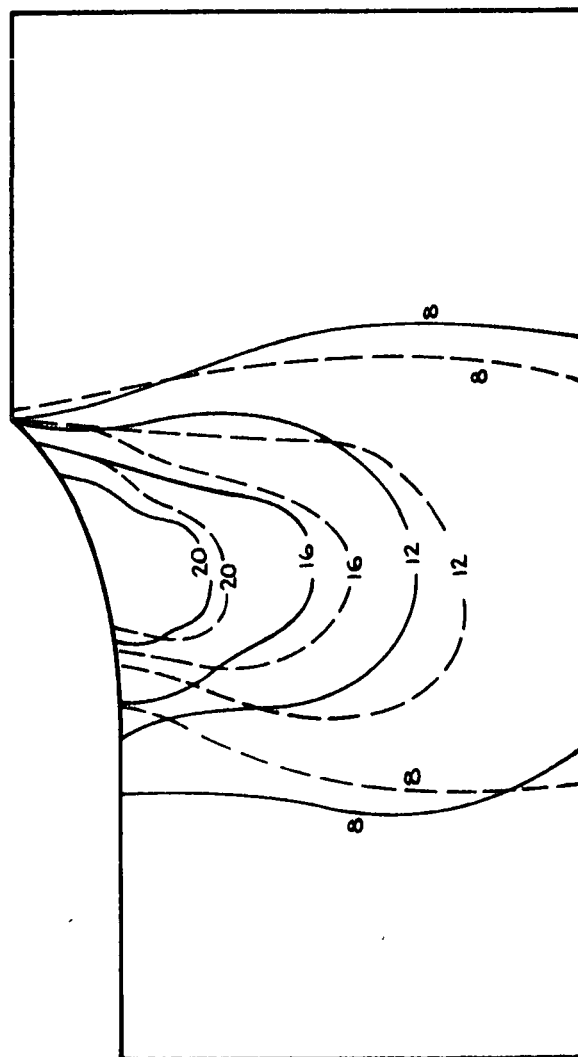


Figure 10: Isobars of Major Principal Stress (in psi)
 for 3.1% Slip

--- Reference 12
 --- Static Elastoplastic Analysis
 (Run ID: WSP - 3)

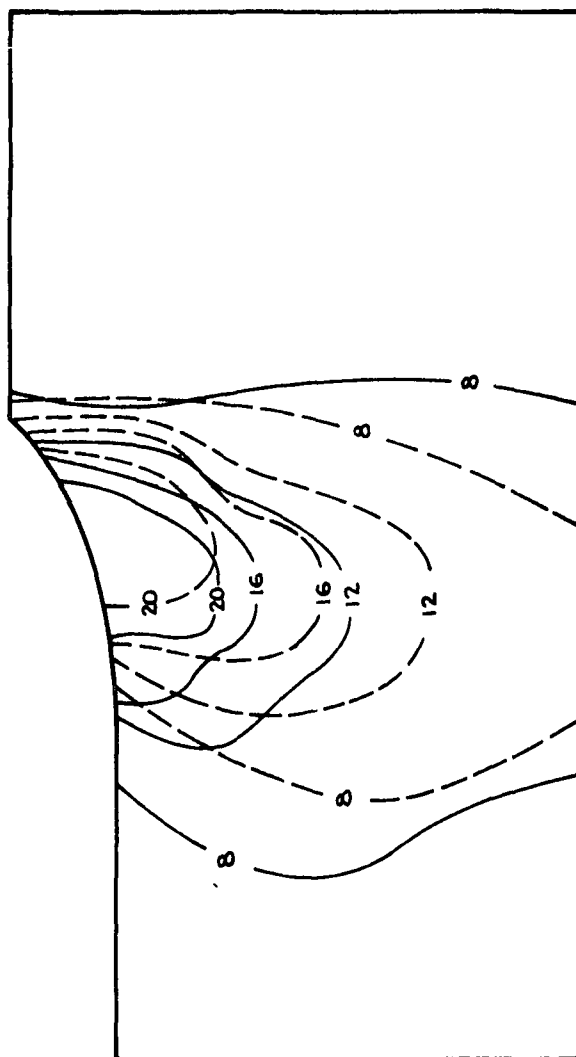


Figure 11: Isobars of Major Principal Stress (in psi)
 for 41.4% Slip

--- Reference 12
 --- Static Plastic Analysis
 (Run ID: WSP - 2)

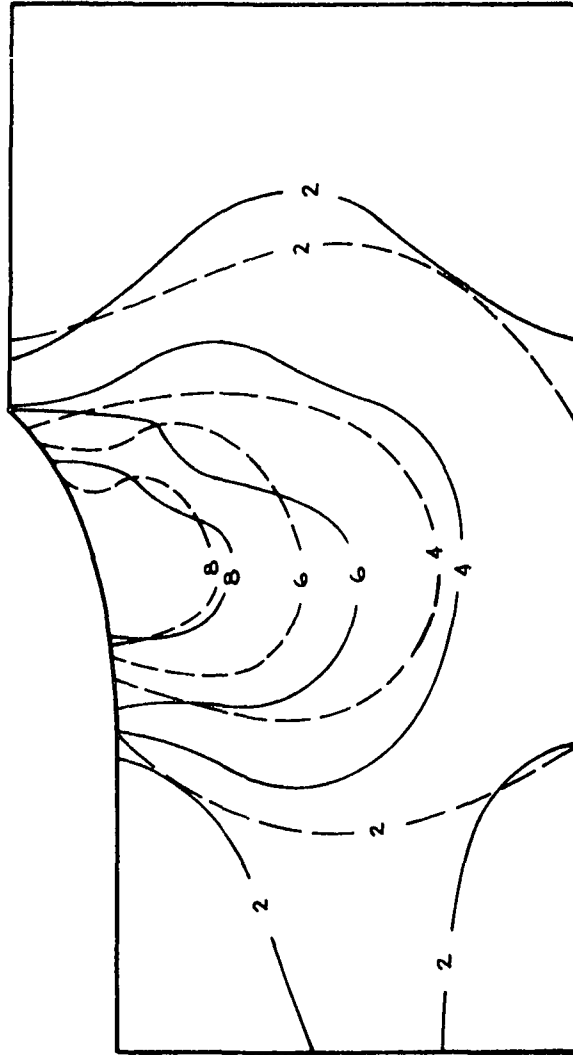


Figure 12: Isobars of Maximum Shear Stress
 for 3.1% Slip

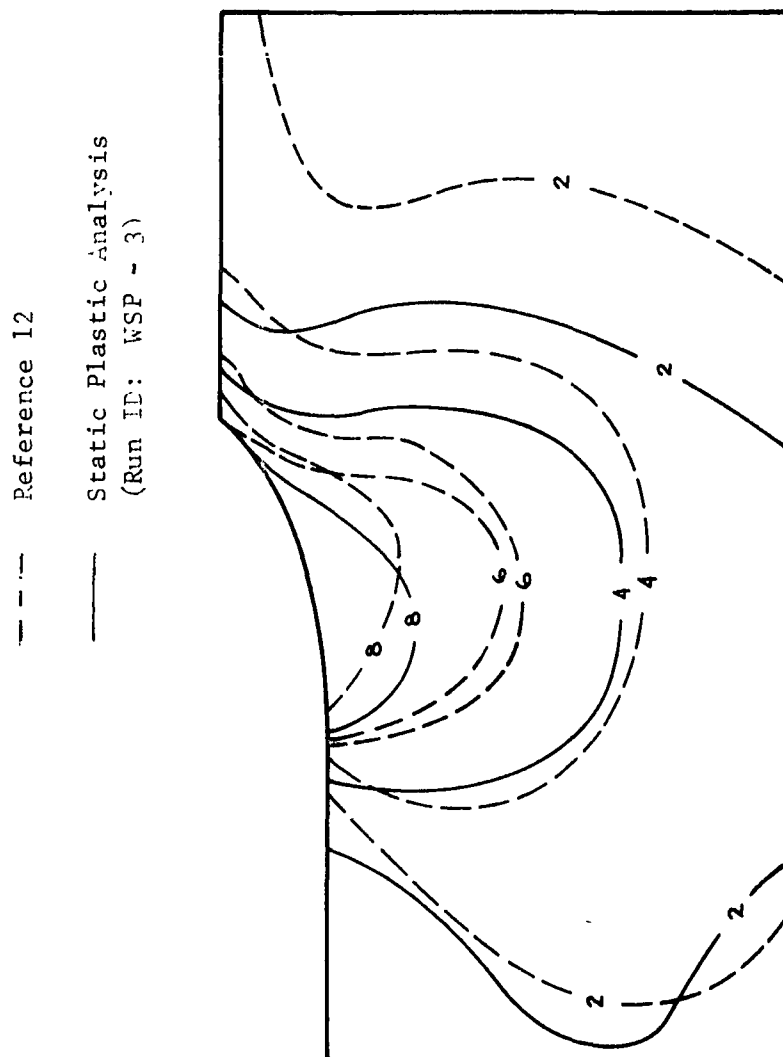


Figure 13: Isobars of Maximum Shear Stress
for 41.4% Slip

--- Maximum Shear Stress
— Major Principal Stress

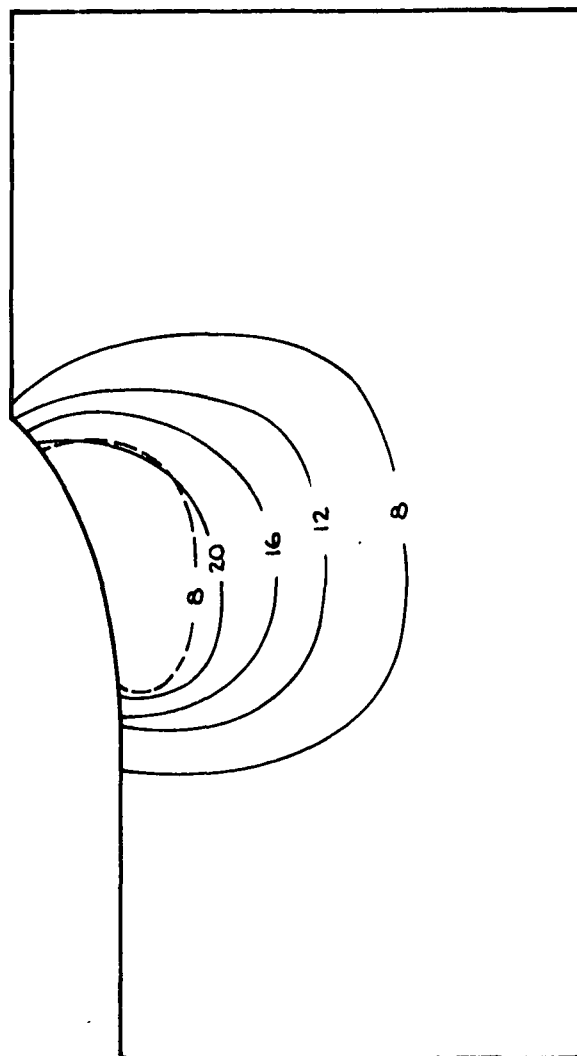


Figure 14: Isobars of Major Principal Stresses
and Maximum Shear Stresses (psi) at
 $t = 0.072$ sec.
(Dynamic Elasto-plastic Analysis for 3.1% Slip)

--- Maximum Shear Stress
 — Major Principal Stress

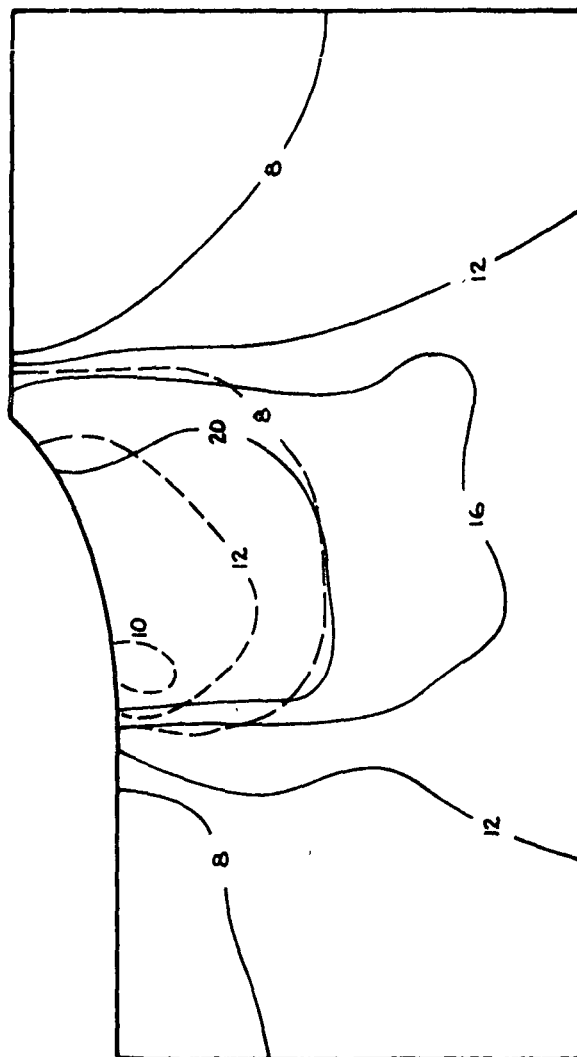


Figure 15: Isobars of Major Principal Stress and Maximum Shear Stress (Psi) at $t=0.15$ sec. (Dynamic Elastoplastic Analysis for 3.1% Slip)

--- Maximum Shear Stress
 --- Major Principal Stress

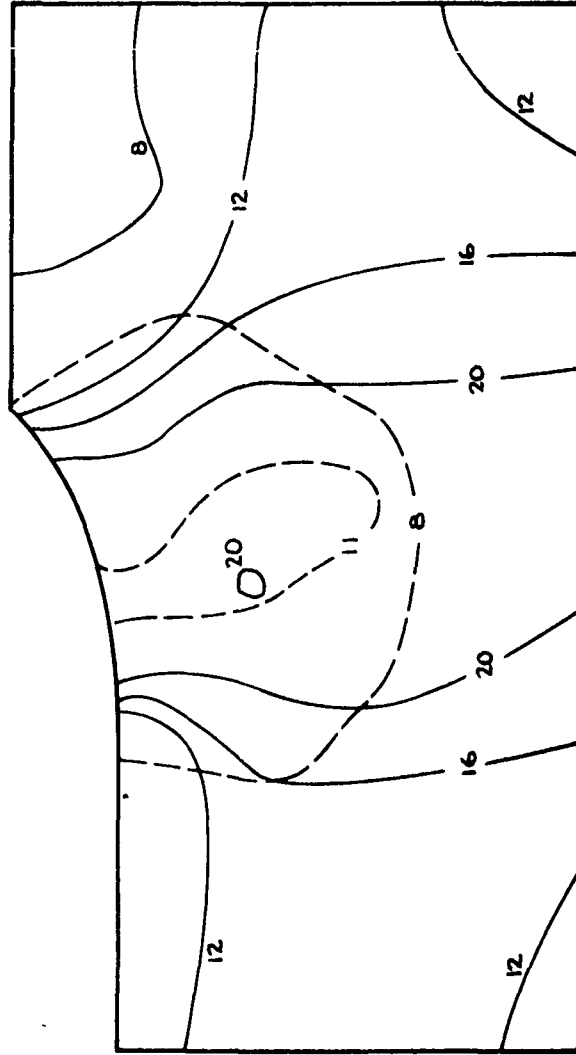


Figure 16: Isobars of Principal Stress and Maximum Shear Stress (Psi) at $t=0.228$ sec. (Dynamic Elastoplastic Analysis for 3.1% slip)

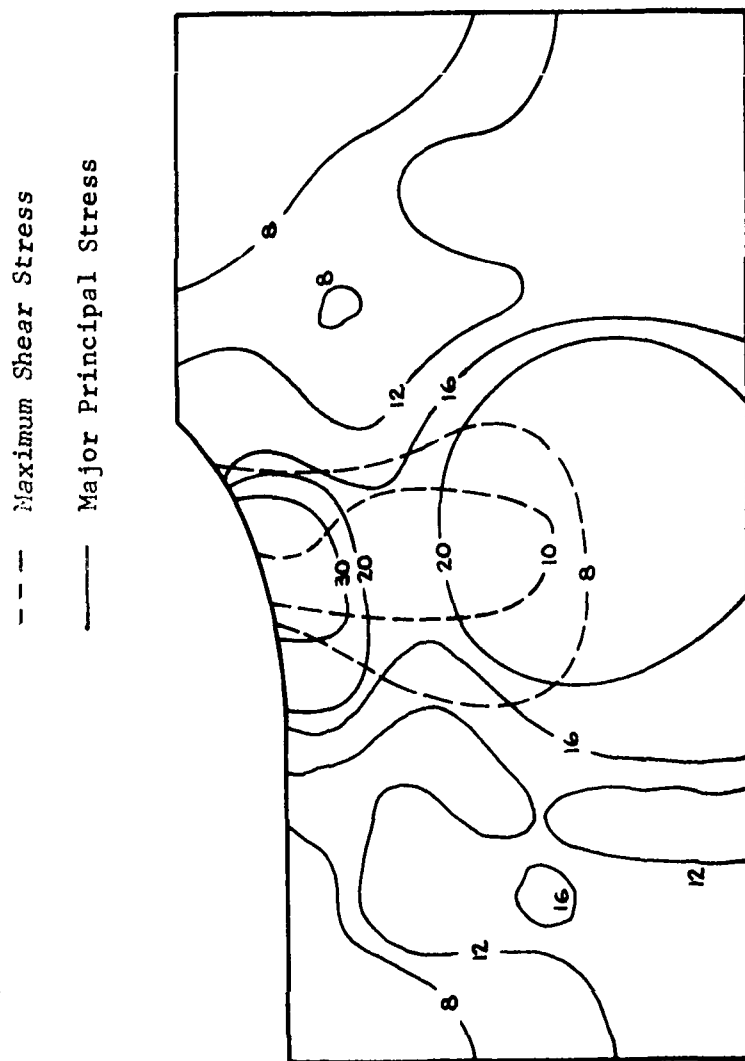


Figure 17: Isobars of Major Principal Stress at Maximum Shear Stress (psi) at $t=0.3$ sec. (Dynamic Elastoplastic Analysis for 3.1% slip)

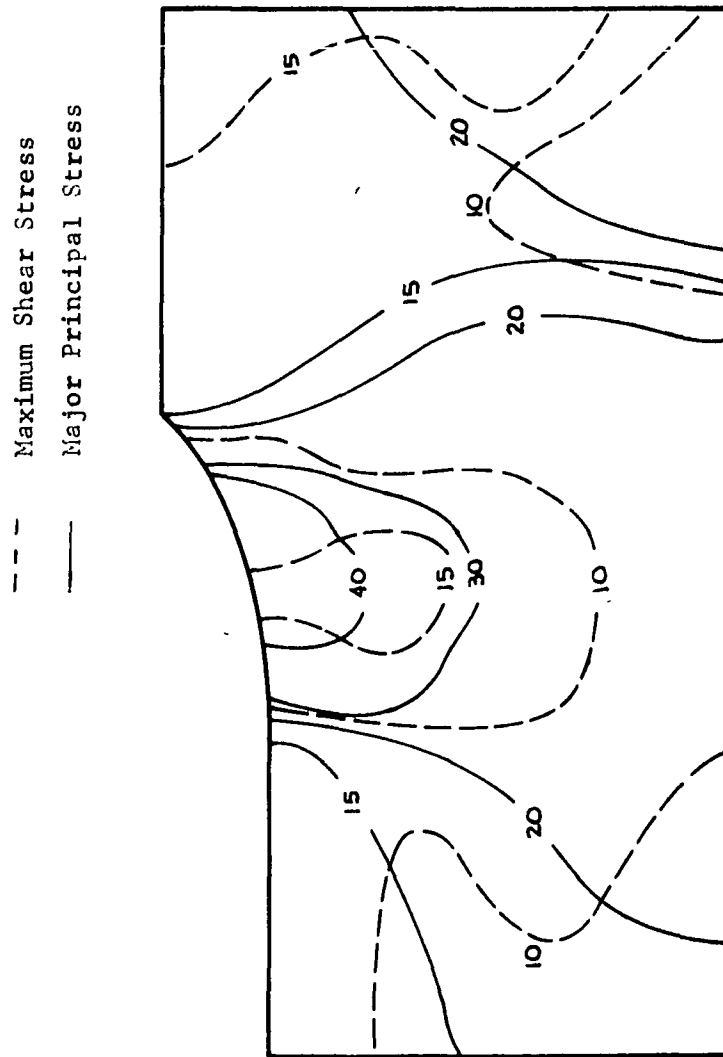


Figure 18: Isobars of Major Principal Stresses and Maximum Shear Stresses
at $t=0.3$ sec (Dynamic Viscoelastoplastic Analysis for 3.1% Slip)

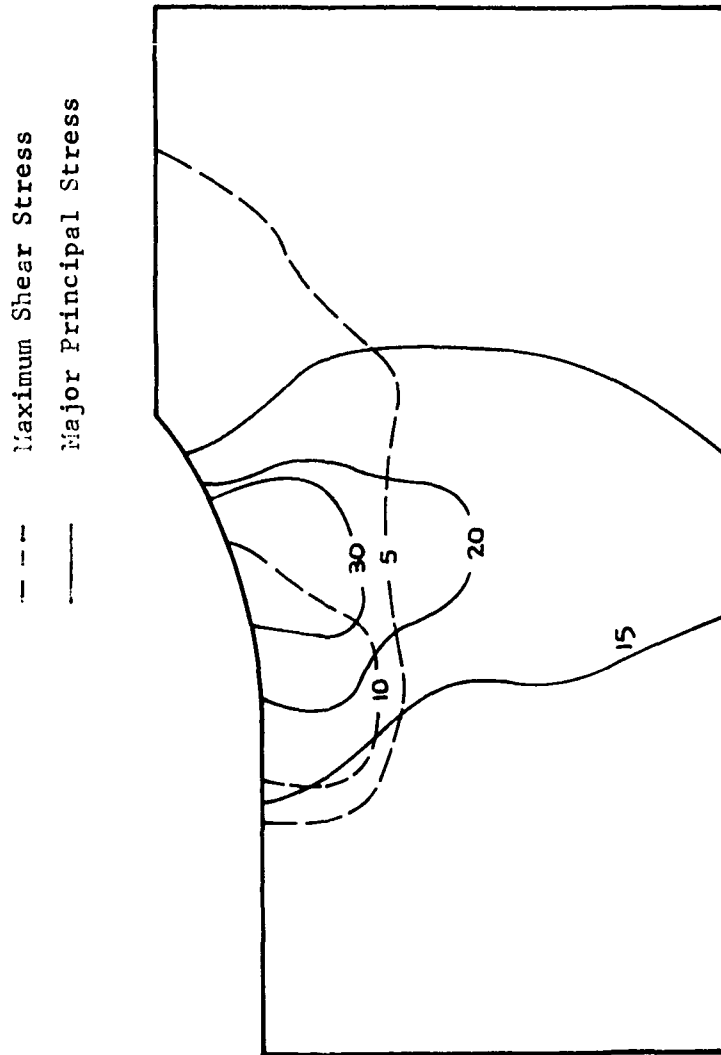


Figure 19: Isobars of Major Principal Stresses and Maximum Shear Stresses (psi) at .6 sec (Dynamic Viscoelastoplastic Analysis for 3.1% Slip)

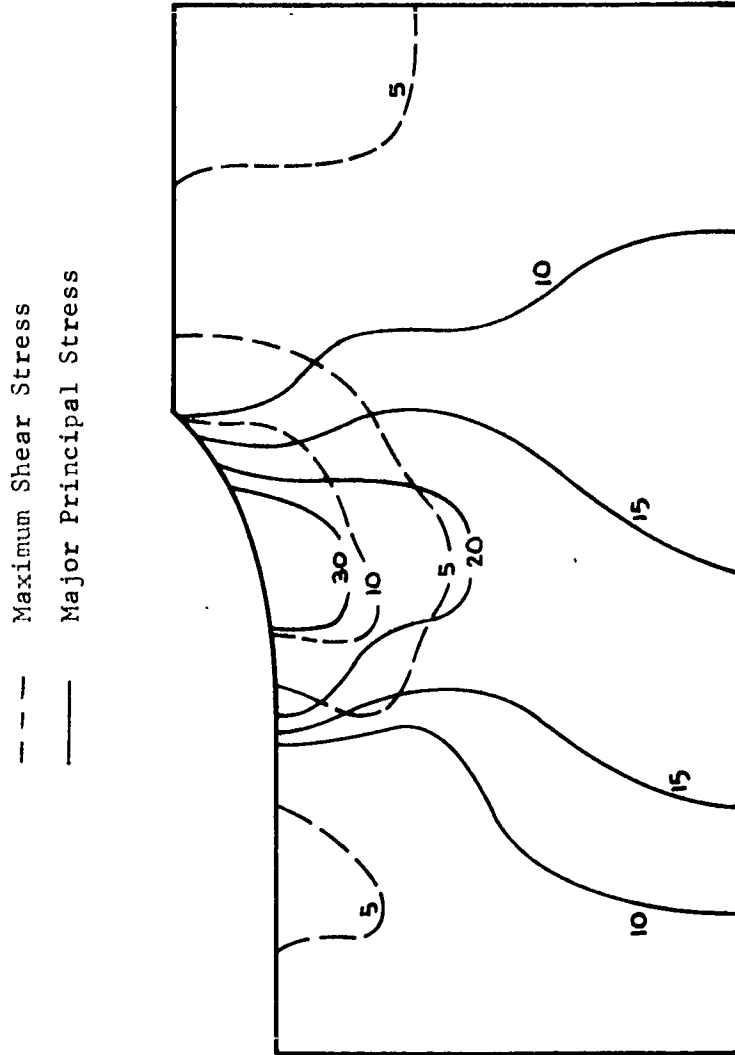


Figure 20: Isobars of Maximum Shear and Major Principal Stresses (psf) at $t=0.3$ sec. (Dynamic Viscoelastoplastic Analysis for 41.4% Slip)

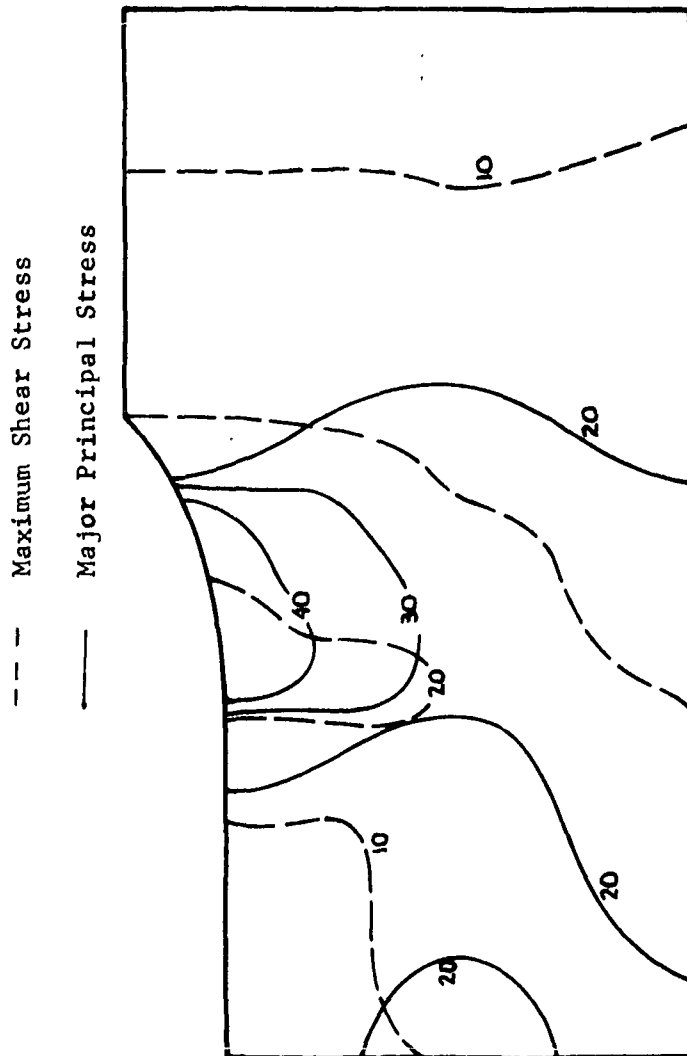


Figure 21: Isobars of Major Principal Stress and Maximum Shear Stress (Psi) at $t=0.65$ sec (Dynamic Viscoelastoplastic Analysis for 41.4% Slip)

interaction cannot be understood properly if incorrect judgement or oversimplification in the theoretical formulation obscures the true deformation and stress fields. For this reason, the present study was devoted to a new approach in which rate-dependent inelastic behavior coupled with effects of inertia was considered.

The analysis presented in the previous sections becomes the stepstone for characterizing the soil mechanics parameters more realistically. Of course, all the results obtained here are based on hypothetical material constants. However, if the analytical formulations are correct, then the wheel-soil interaction data as observed qualitatively and quantitatively may be used to correlate with material constants. Such characterization can be achieved by holding some of the material parameters constant and comparing the load-deformation data between the calculated and observed values.

Because the present study does not include the dust cloud motion behind the lunar rover the observed rooster-tailing cannot be related to the material characterization. However, the sinkage of the rover wheel together with the vehicle performance data can be used for correlation with deformation and stress fields as mentioned in the previous paragraph.

II-10. CONCLUSIONS

The main objective of the present study was to introduce a feasible constitutive relationship for soil deformation and stress fields under a moving wheel. The load transmitted by the moving wheel is dynamic rather

than static. The soil is dissipative media in which inelastic deformation of the soil is governed by the rate-dependent plasticity or viscoelastoplasticity. The yield surface theory of Roscoe and Burland is utilized here for inelastic behavior. The internal variables are then introduced to account for rate-dependent viscous behavior. Effects of soil inertia are included. Combinations of all of these properties result in dynamic analysis of viscoelastoplastic media.

The numerical results obtained here appear very reasonable. Comparisons with the results of other investigators are made and deviations are believed to be due to more rigorous treatment of material behavior considered in the present study. In order to verify the impact of the theoretical formulations given here, however, additional comparison study through experimental data is needed.

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APPENDIX 1

DERIVATION OF INTERNAL (HIDDEN) VARIABLES

Consider the internal variable $\alpha_{ij}^{(r)}$

$$\alpha_{ij}^{(r)}(t) = \int_0^t \exp\left(\frac{-(t-\tau)}{T_{(r)}}\right) \dot{\gamma}_{ij}(\tau) d\tau \quad (A-1)$$

where $\dot{\gamma}_{ij}(\tau)$ may be considered to vary linearly within the small time interval Δt ,

$$\dot{\gamma}_{ij}(\tau) = \dot{\gamma}_{ij}(t-\Delta t) + \frac{\tau-(t-\Delta t)}{\Delta t} [\dot{\gamma}_{ij}(t) - \dot{\gamma}_{ij}(t-\Delta t)] \quad (A-2)$$

Substituting (A-2) in (A-1),

$$\begin{aligned} \alpha_{ij}^{(r)}(t) &= \int_0^{t-\Delta t} \exp\left(\frac{-(t-\tau)}{T_{(r)}}\right) \dot{\gamma}_{ij}(\tau) d\tau + \int_{t-\Delta t}^t \exp\left(\frac{-(t-\tau)}{T_{(r)}}\right) \dot{\gamma}_{ij}(\tau) d\tau \\ &= \exp\left(\frac{-\Delta t}{T_{(r)}}\right) \alpha_{ij}^{(r)}(t-\Delta t) + \int_{t-\Delta t}^t \exp\left(\frac{-(t-\tau)}{T_{(r)}}\right) \dot{\gamma}_{ij}(\tau) d\tau \\ &= \exp\left(\frac{-\Delta t}{T_{(r)}}\right) \alpha_{ij}^{(r)}(t-\Delta t) + \int_{t-\Delta t}^t \exp\left(\frac{-(t-\tau)}{T_{(r)}}\right) \left\{ \dot{\gamma}_{ij}(t-\Delta t) \right. \\ &\quad \left. + \frac{\Delta t - t + \tau}{\Delta t} [\dot{\gamma}_{ij}(t) - \dot{\gamma}_{ij}(t-\Delta t)] \right\} d\tau \end{aligned}$$

$$\begin{aligned}
&= \exp\left(\frac{-\Delta t}{T(r)}\right) \alpha_{ij}^{(r)}(t-\Delta t) + \int_{t-\Delta t}^t \exp\left(\frac{-(t-\tau)}{T(r)}\right) \left\{ \left(1 - \frac{t}{\Delta t} + \frac{\tau}{\Delta t}\right) \dot{\gamma}_{ij}(\tau) \right. \\
&+ \left. \left(\frac{t}{\Delta t} - \frac{\tau}{\Delta t}\right) \dot{\gamma}_{ij}(t-\Delta t) \right\} d\tau = \exp\left(\frac{-\Delta t}{T(r)}\right) \alpha_{ij}^{(r)}(t-\Delta t) + T(r) \left\{ \left[\exp\left(\frac{-(t-\tau)}{T(r)}\right) \right. \right. \\
&- \frac{t}{\Delta t} \exp\left(\frac{-(t-\tau)}{T(r)}\right) + \frac{\tau}{\Delta t} \exp\left(\frac{-(t-\tau)}{T(r)}\right) - \frac{T(r)}{\Delta t} \exp\left(\frac{-(t-\tau)}{T(r)}\right) \left. \right]_{t-\Delta t}^t \dot{\gamma}_{ij}(\tau) \\
&+ \left. \left[\frac{t}{\Delta t} \exp\left(\frac{-(t-\tau)}{T(r)}\right) - \frac{\tau}{\Delta t} \exp\left(\frac{-(t-\tau)}{T(r)}\right) + \frac{T(r)}{\Delta t} \exp\left(\frac{-(t-\tau)}{T(r)}\right) \right]_{t-\Delta t}^t \dot{\gamma}_{ij}(t-\Delta t) \right\} \\
&= \exp\left(\frac{-\Delta t}{T(r)}\right) \alpha_{ij}^{(r)}(t-\Delta t) + T(r) \left[\left\{ 1 - \exp\left(\frac{-\Delta t}{T(r)}\right) - \frac{t}{\Delta t} \left[1 - \exp\left(\frac{-\Delta t}{T(r)}\right) \right] \right. \right. \\
&+ \left. \frac{t}{\Delta t} - \frac{t-\Delta t}{\Delta t} \exp\left(\frac{-\Delta t}{T(r)}\right) - \frac{T(r)}{\Delta t} + \frac{T(r)}{\Delta t} \exp\left(\frac{-\Delta t}{T(r)}\right) \right\} \dot{\gamma}_{ij}(t) \\
&+ \left\{ \frac{t}{\Delta t} \left[1 - \exp\left(\frac{-\Delta t}{T(r)}\right) \right] - \frac{t}{\Delta t} + \frac{t-\Delta t}{\Delta t} \exp\left(\frac{-\Delta t}{T(r)}\right) + \frac{T(r)}{\Delta t} \right. \\
&- \left. \frac{T(r)}{\Delta t} \exp\left(\frac{-\Delta t}{T(r)}\right) \right\} \dot{\gamma}_{ij}(t-\Delta t) \left. \right] = \exp\left(\frac{-\Delta t}{T(r)}\right) \alpha_{ij}^{(r)}(t-\Delta t) \\
&+ T(r) \left[\left\{ -\exp\left(\frac{-\Delta t}{T(r)}\right) + \frac{T(r)}{\Delta t} \left[1 - \exp\left(\frac{-\Delta t}{T(r)}\right) \right] \right\} \dot{\gamma}_{ij}(t-\Delta t) \right. \\
&+ \left. \left\{ 1 - \frac{T(r)}{\Delta t} \left[1 - \exp\left(\frac{-\Delta t}{T(r)}\right) \right] \right\} \dot{\gamma}_{ij}(t) \right] = \alpha_{ij}^{(r)}(t-\Delta t) \\
&+ \dot{B}^{(r)} \dot{\gamma}_{ij}(t-\Delta t) + \dot{C}^{(r)} \dot{\gamma}_{ij}(t)
\end{aligned}$$

or

$$\alpha_{ij}^{(r)}(s) = \tilde{A}^{(r)} \alpha_{ij}^{(r)}(s-1) + \tilde{B}^{(r)} \dot{y}_{ij}(s-1) + \tilde{C}^{(r)} \dot{y}_{ij}(s)$$

where

$$\tilde{A}^{(r)} = \exp \frac{-\Delta t}{T_{(r)}}$$

$$\tilde{B}^{(r)} = T_{(r)} \left[\overset{\sim}{\Phi}^{(r)} - \tilde{A}^{(r)} \right]$$

$$\tilde{C}^{(r)} = T_{(r)} [1 - \overset{\sim}{\Phi}^{(r)}]$$

$$\overset{\sim}{\Phi}^{(r)} = \frac{T_{(r)}}{\Delta t} (1 - \tilde{A}^{(r)})$$

APPENDIX 2

CONTACT STRESSES AT WHEEL-SOIL INTERFACE

The vertical and horizontal forces and torque of a wheel rotating on horizontal ground with constant velocity are given by

$$\begin{aligned}
 W &= rb \left\{ \int_{\theta_2}^{\theta_1} \sigma(\theta) \cos \theta d\theta + \int_{\theta_2}^{\theta_1} \tau(\theta) \sin \theta d\theta \right\} \\
 D &= rb \left\{ \int_{\theta_2}^{\theta_1} \tau(\theta) \cos \theta d\theta - \int_{\theta_2}^{\theta_1} \sigma(\theta) \sin \theta d\theta \right\} \\
 T &= r^2 b \int_{\theta_2}^{\theta_1} \tau(\theta) d\theta
 \end{aligned}$$

in which $\sigma(\theta)$ and $\tau(\theta)$ are the average radial and tangential stress across the wheel width of b (Fig. 2-1).

The location of the point of the maximum radial stress may be expressed as

$$\theta_M = (C_1 + C_2 s) \theta_1$$

where s is the slip (%) defined by

$$s = \left(1 - \frac{V}{\omega r}\right) 100$$

and C_1 and C_2 are the constants [14-17] given in Table 1.

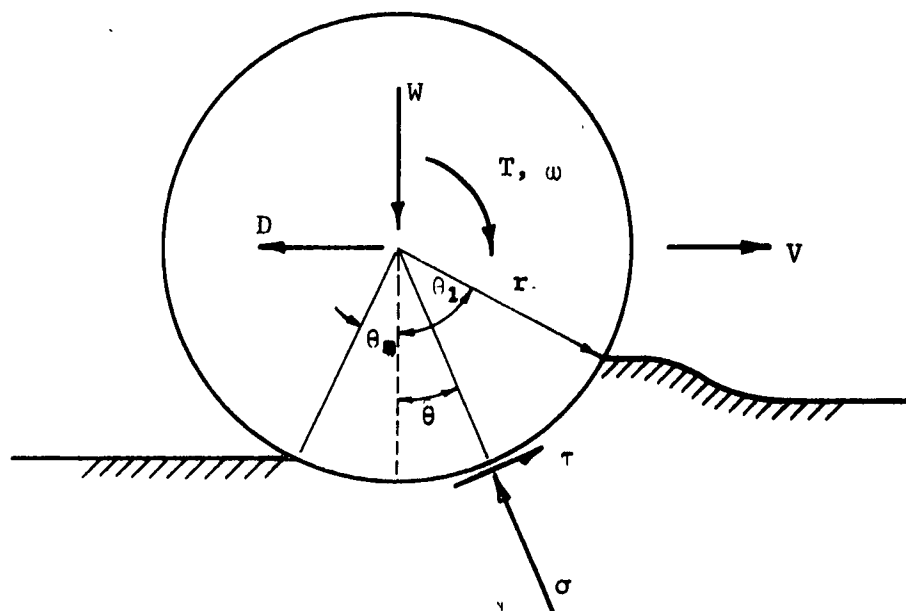


Figure 2-1: Equilibrium of a Driven Rigid Wheel on Soil

TABLE 1

Soil	Angle of Internal Friction (deg.)	Soil Cohesion	Density (lb/in ³)	C ₁	C ₂	K ₁	K ₂	n	Shear Deformation Modulus (in)
Compact sand	33.3	0.10	0.0575	0.413	0.32	20	2.5	0.47	1.5
Loose sand	31.1	0.12	0.048	0.18	0.32	0	2	1.15	1.5
Sand	36.0	0.10	0.0617	0.285	0.32	-	-	-	-
Dry sand	24.0	-	-	0.38	0.41	-	-	-	-

In the region between θ_1 and θ_m or the front region, the radial stress is given by [18]

$$\sigma_1(\theta) = (K_1 + K_2 b) \left(\frac{r}{6}\right)^n (\cos \theta - \cos \theta_1)^n$$

where the constants K_1 , K_2 , and n are shown in Table 1. The radial stress acting in the rear region is of the form

$$\sigma_2(\theta) = (K_1 + K_2 b) \left(\frac{r}{6}\right)^n \left[\cos \left\{ \theta_1 - \theta \left(\frac{C_1 + C_{21}}{C_1 + C_{21}} \right) \right\} - \cos \theta_1 \right]^n$$

The shear stress around the rim is given by [14,15],

$$\tau(\theta) = (C + \sigma(\theta) \tan \phi) (1 - e^\beta)$$

where C is the cohesion, and

$$\beta = \frac{-r}{K} \{ (\theta_1 - \theta) - (1-i)(\sin \theta_1 - \sin \theta) \}$$

In the above expressions θ_1 is still not known but can be determined from the expression of the vertical force W,

$$W = rh \left\{ \int_{\theta_M}^{\theta_1} \sigma_1(\theta) \cos \theta d\theta + \int_0^{\theta_M} \sigma_2(\theta) \cos \theta d\theta \right. \\ \left. + \int_{\theta_M}^{\theta_1} \tau_1(\theta) \sin \theta d\theta + \int_0^{\theta_M} \tau_2(\theta) \sin \theta d\theta \right.$$

where,

$$\tau_1(\theta) = \left(C + \sigma_1(\theta) \tan \phi \right) (1 - e^{\beta})$$

$$\tau_2(\theta) = \left(C + \sigma_2(\theta) \tan \phi \right) (1 - e^{\beta})$$

If the magnitude of W is given then the above integration may be carried out by the Simpson's rule and θ_1 is solved in terms of known values.

With the value of θ_1 known, we can then calculate the radial and tangential stresses.

Finally, the wheel sinkage z_0 is determined from

$$z_0 = (1 - \cos \theta_1) r$$

APPENDIX 3

COMPUTER PROGRAM LISTING

(Dynamic Wheel-Soil Interaction, Plane Strain)

```

00001000 MSG,T 2,F/1/PUS75
00001000
00001000 C ISW(1) = 0, STATIC ANALYSIS ONLY 00000100
00001000 C ISW(1) = N, DYNAMIC ANALYSIS FOR N TIME INCREMENTS. 00000200
00001000 C ISW(1) = -N, STATIC PLASTIC ANALYSIS FOR N LOAD INCREMENT 00000300
00001000 C ISW(2) = -1, VISCO-ELASTIC 00000400
00001000 C ISW(2) = 0, ELASTIC 00000500
00001000 C ISW(2) = 1, VISCO-ELASTO-PLASTIC 00000600
00001000 C ISW(3) = M, PRINT FOR EACH M TH TIME STEP 00000700
00001000 PARAMETER NF=150,NELS=150,MX= 5000,NF=NFT*2 00000800
00001000 COMMON /BLK0/ TITLE(20),INODE,NELEM,NAPC,NBC,MAT1,ISW(5) 00000900
00001000 COMMON /BLK1/ W(4),F(4),AR(4),BR(4),CR(4),A7(4),BZ(4),CZ(4), 00001000
00001000 * FN(4),CN(4),DN(4),TYPEA(4,4),TYPEB(4,4),TYPEC(4,4),TYPEU(4,4), 00001100
00001000 * AU,BU,CU,IC,UC,KC,IC,NEL 00001200
00001000 COMMON /BLK2/ ID(4,4,2),TJKL(NELS,4),R(NFT),Z(NFT),KK(NELS,8) 00001300
00001000 COMMON /BLK3/ AKL(MX),P(NF),IMAX,IHR,IHRI,LT,IAST,NFRFE 00001400
00001000 COMMON /BLK4/ U(3,3),AP(3,3),AD(3,3),BD(3,3),STIFF(8,8),CM(8,8), 00001500
00001000 * VE(8,8),A1(NELS) 00001600
00001000 COMMON /BLK5/ GR(NELS,3),STRAIN(NELS,3),ALPHA,BETA,GAMMA,DELT,XK, 00001700
00001000 * AT,XNU,E,SM,SMS,VOIDI,CAPA,RAMD,BLT 00001800
00001000 COMMON /BLK6/ DP(NELS,3,3),PRINS(NELS,3) 00001900
00001000 COMMON /DYN/ CDAR(MX),XV(MX),AA(MX) 00002000
00001000 C 00002100
00001000 C 00002200
00001000 CALL SETUP 00002300
00001000 C 00002400
00001000 READ(5,510) (TITLE(I),I=1,20) 00002500
00001000 READ(5,500) INODE,NELEM,NAPC,NBC 00002600
00001000 READ(5,500) ISW 00002700
00001000 READ(5,530) E,XNU,DENS,DEPTH,ES,XNUS 00002800
00001000 READ(5,530) AT,XK,DELT 00002900
00001000 READ(5,530) PHI,VOIDI,CAPA,RAMD 00003000
00001000 PHI = PHI * 3.14159 / 180. 00003100
00001000 SINP = SIN(PHI) 00003200
00001000 SM = 6.*SINP / (3.-SINP) 00003300
00001000 SMS = SM * SM 00003400
00001000 BET = RAMD - CAPA 00003500
00001000 WRITE(6,630) INODE,NELFM,NAPC,NBC 00003600
00001000 WRITE(6,610) E,XNU,DENS,AT,XK,DELT 00003700
00001000 WRITE(6,640) PHI,VOIDI,CAPA,RAMD 00003800
00001000 II = 0 00003900
00001000 DO 101 I = 1,INODE 00004000
00001000 DO 101 J = 1,2 00004100
00001000 II = II + 1 00004200
00001000 101 ID(I,J) = II 00004300
00001000 WRITE(6,600) (TITLE(I),I=1,20) 00004400
00001000 DO 100 I = 1,INODE 00004500
00001000 READ(5,520) Z(1),R(1),IZ,IR 00004600
00001000 IF(IZ.NE.0) ID(I,1) = 0 00004700
00001000 IF(IR.NE.0) ID(I,2) = 0 00004800
00001000 100 WRITE(6,620) I,Z(1),R(1),IZ,IR 00004900
00001000 C 00005000
00001000 C 00005100
00001000 CALL INIT 00005200
00001000 NBC = INODE 00005300

```


000	NFREE = INODE * 2	00005400
000	NACT = NFREE	00005500
000	C	00005600
000	WRITE(6,650)	00005700
000	READ(5,550) ((IJKL(I,J),J=1,4),I=1,NELEM)	00005800
000	WRITE(6,660) (1,(IJKL(1,J),J=1,4),1=1,NELEM)	00005900
000	DO 500 NEL = 1,NELEM	00006000
000	DO 700 K = 1,4	00006100
000	IN = IJKL(NEL,K)	00006200
000	KK(NEL,K) = ID(IN,1)	00006300
000	700 KK(NEL,K+4) = ID(IN,2)	00006400
000	NELI = ISW(5) + 1	00006500
000	E = 25	00006600
000	XNU = XNUS	00006700
000	IF(NEL.EQ.NELI) CALL INIT	00006800
000	GO 750 1 = 1,3	00006900
000		
000		
000	DO 790 J = 1,3	00007000
000	790 UF(NEL,I,J) = U(1,J)	00007100
000	DO 799 I = 1,2	00007200
000	II = J + 1	00007300
000	DO 799 J = II,4	00007400
000	IDIF = IJKL(NEL,1) - IJKL(NEL,J)	00007500
000	IF(IDIF.LT.0) IDIF = -IDIF	00007600
000	799 IF(IDIF.GT.IMAX) IMAX = IDIF	00007700
000	800 CONTINUE	00007800
000	C IMAX = MAX DIFFERENCE IN ADJACENT NODE NO.	00007900
000	IHB = (IMAX + 1) * 2	00008000
000	IHBI = IHB - 1	00008100
000	LT = IHB * IHB1 / 2	00008200
000	LAST = LI + (NFREE - IHB1) * IHB	00008300
000	WRITE(6,641) IMAX,IHB,LT,LAST	00008400
000	C	00008500
000	WRITE(6,677)	00008600
000	DO 920 NC = 1,NAPC	00008700
000	READ(5,540) NODE,PZ,PR	00008800
000	WRITE(6,688) NODE,PZ,PR	00008900
000	II = (NODE-1) * 2	00009000
000	P(II+1) = PZ	00009100
000	P(II+2) = PR	00009200
000	920 CONTINUE	00009300
000	IF(ISW(1).GT.0) CALL DIAGNL(XM,P,NFREE,IHB,IHBI,LT,LAST)	00009400
000	IF(ISW(1).LE.0) CALL DIAGNL(XKL,P,NFREE,IHB,IHBI,LT,LAST)	00009500
000	C	00009600
000	CALL STIFF1(DENS)	00009700
000	C	00009800
000	WRITE(6,693) D	00009900
000	C	00010000
000	C	00010100
000	IF(ISW(1).GT.0) GO TO 930	00010200
000	C	00010300
000	CALL ZERO(QB,NELS,3)	00010400
000	IF(ISW(1).LT.0) CALL PLASTC	00010500
000	C	00010600
000	CALL DISPL(1)	00010700

```

000      CALL STRAIN(1)                                00010900
000      STOP ELASTC                                    00010900
000      C                                              00011000
000      C                                              00011100
000      930 CONTINUE                                  00011200
000      C                                              00011300
000      CALL STEP                                       00011400
000      C                                              00011500
000      C                                              00011600
000      1000 STOP                                       00011700
000      C                                              00011800
000      500 FORMAT(10I5)                               00011900
000      510 FORMAT(20A4)                               00012000
000      520 FORMAT(5X,2F10.4,2I5)                     00012100
000      530 FORMAT(6F15.6)                             00012200
000      540 FORMAT(15,2F10.4)                         00012300
000      550 FORMAT(5X,4I5)                             00012400
000      600 FORMAT(1H1,20X,20A4//50X,'COORDINATE VALUES'//T11,'NODE',I30,'7-CORD',I15,
000      *CORD',I15,'K-CORD',I15,'0,IF FREE TO Z',3X,'0,IF FREE TO R'//) 00012500
000      610 FORMAT(//5X,'E',XND,' DENS, AT, XK, DELT',,6E15.7//) 00012700
000      620 FORMAT(110,I5,I25,F10.4,I45,F10.4,I65,2(18,5X)) 00012800
000      630 FORMAT(//5X,'INODE, NLELM, NAPL, NRC',,4I5) 00012900
000      640 FORMAT(5X,'PHI, VOIDI, CAPA, RAMDA',,4E15.6) 00013000
000      641 FORMAT(//5X,'IMAX =',I5,' IHB =',I5,' LT =',I5,' LAST =',I5) 00013100
000      650 FORMAT(//5X,'CONNECTIVITY'//)              00013200
000      660 FORMAT(5I8)                                00013300
000      677 FORMAT(///10X,'APPLIED (, LOAD'//5X,'NODE',I10X,'FORCE TO Z',5X,'
000      * FORCE TO R'//)                                00013500
000      688 FORMAT(5X,I5,2(5X,F12.4))                  00013600
000      689 FORMAT(1H1,10X,' TOTAL DISPLACEMENT      NO. OF INCREMENTAL STEPS
000      * =',2I5//5X,'NODE',5X,'Z - DISPL',20X,'R - DISPL'//) 00013700
000      690 FORMAT(5X,I5,2E15.7)                      00013800
000      691 FORMAT(///10X,' TOTAL STRESSES'//5X,'ELEM',5X,'SIGMA - Z',10X,
000      *SIGMA - R',10X,'TANGENTIAL',10X,'TAU - ZR'//) 00013900
000      692 FORMAT(5X,I5,3X,4F17.6)                  00014000
000      693 FORMAT(3E15.6)                            00014100
000      END                                             00014200
000      INFL,SIH NASA*IPFS,INIT,,16J532132410
000      SUBROUTINE INIT                                00000100
000      PARAMETER NF1=150,NF2=150,MX=5000,NF=NF1*2    00000200
000      COMMON /BLK4/ D(3,3),ATD(3,3),AD(3,3),BD(3,3),STIFF(8,8),CM(8,8), 00000300
000      * VE(8,8),A1(NELS)                             00000400
000      COMMON /BLK5/ GP(NELS,3),STPAIR(NELS,3),ALPHA,BETA,GAMMA,DELT,XK, 00000500
000      * AT,XMU,E,SM,SMS,VOIDI,CAPA,RAMD,BET         00000600
000      DIMENSION GD(3,3),DV(3,3),ATA(3,3)           00000700
000      CONST = E*XMU / ((1.+XMU)*(1.-XMU*2.))         00000800
000      SHEAR = E / (2.*(1.+XMU))                     00000900
000      D(1,1) = CONST + SHEAR*2.                     00001000
000      D(2,2) = D(1,1)                               00001100
000      D(1,2) = CONST                                00001200
000      D(2,1) = CONST                                00001300
000      D(3,3) = SHEAR                                00001400
000      TI = AT/XK                                     00001500
000      LSUM = 3                                       00001600
000      CALL ZERO(ATA,3,3)                             00001700
000      CALL ZERO(DV,3,3)                             00001800
000      LU 50 1 = 1,3                                00001900

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000      50 DV(I,J) = XK
000      ALPHA=0.0
000      BETA=0.0
000      GAMMA=0.0
000      IF (ALC(11).LT.0.000001) GO TO 20
000      DT = -DEL1/11
000      ALPHA = EXP(DT)
000      BETA = 11*(-ALPHA-(1.-ALPHA)/DT)
000      GAMMA = 11*(1.+(1.-ALPHA)/DT)
000      20 CONTINUE
000      DO 6 I=1,3
000      DO 6 J=1,3
000      AD(I,J)= 0.
000      HD(I,J)=0.
000      GD(I,J)=0.
000      DO 7K=1,LSUM
000      AD(I,J)= AD(I,J) +DV (I,J)*ALPHA
000      HD(I,J)= HD(I,J)+DV (I,J)*BETA
000      7 GD(I,J)=GD(I,J)+DV (I,J)*GAMMA
000      DO 8 L=1,3
000      8 AD(L,L)=AD(L,L)+GD(L,L)
000      6 CONTINUE
000      RETURN
000      END
000      16FL7,SIF NASA*IPFS,SIF 1,,165535132410
000      SUBROUTINE STIFF1(DERS)
000      PARAMETER NF=150,NELS=150,MX= 5000,NF=NFT*2
000      COMMON /BLK0/ IITLE(20),INODE,NFLEN,NAPC,NHC,MAC1,ISW(5)
000      COMMON /BLK1/ W(4),H(4),AR(4),BR(4),CR(4),AZ(4),BZ(4),CZ(4),
000      * BN(4),CN(4),DN(4),TYPEA(4,4),TYPEB(4,4),TYPEC(4,4),TYPEF(4,4),
000      * AU,BU,CU,IC,JC,KC,LC,NEL
000      COMMON /BLK2/ IDINF(2),IJKL(NELS,4),K(NF),Z(NFT),KK(NELS,8)
000      COMMON /BLK3/ XPL(MX),P(NF),IMAX,IHR,IHR1,LT,LAST,NFREE
000      COMMON /BLK4/ D(3,3),AD(3,3),AP(3,3),HD(3,3),STIFF(8,8),CM(8,8),
000      * VE(8,8),AI(NELS)
000      COMMON /BLK6/ BT(NELS,8,3),ARM(NELS,4),AZM(NELS,4),AOJ(NELS)
000      COMMON /BLK8/ UP(NELS,3,3),PRINS(NELS,3)
000      COMMON /BLK9/M,N
000      COMMON /DYN/ CBAR(MX),XM(MX),CC(MX)
000      C
000      C
000      DO 900 NEL = 1,NELLEN
000      C
000      IC = IJKL(NEL,1)
000      JC = IJKL(NEL,2)
000      KC = IJKL(NEL,3)
000      LC = IJKL(NEL,4)
000      C
000      CALL AREA(A(NEL),AREA)
000      AI(NEL) = AREA
000      C
000      AZ(1) = Z(LC) - Z(JC)
000      AZ(2) = Z(IC) - Z(KC)
000      AZ(3) = -AZ(1)
000      AZ(4) = -AZ(2)
000      FZ(1) = Z(KC) - Z(LC)
000      BZ(2) = -BZ(1)

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000	PZ(3) = Z(JC) - Z(IC)	00003300
000	HZ(4) = -PZ(3)	00003400
000	CZ(1) = Z(JC) - Z(KC)	00003500
000	CZ(2) = Z(LC) - Z(IC)	00003600
000	CZ(3) = -CZ(2)	00003700
000	CZ(4) = -CZ(1)	00003800
000	AK(1) = K(JC) - K(LC)	00003900
000	AK(2) = K(KC) - K(IC)	00004000
000	AK(3) = -AK(1)	00004100
000	AK(4) = -AK(2)	00004200
000	HK(1) = K(LC) - K(KC)	00004300
000	HK(2) = -HK(1)	00004400
000	HK(3) = K(IC) - K(JC)	00004500
000	HK(4) = -HK(3)	00004600
000	CK(1) = K(KC) - K(JC)	00004700
000	CK(2) = K(IC) - K(LC)	00004800
000	CK(3) = -CK(2)	00004900
000	CK(4) = -CK(1)	00005000
000	PO 100 I = 1,4	00005100
000	AZ(I) = -AZ(1)	00005200
000	PZ(I) = -PZ(1)	00005300
000	CZ(I) = -CZ(1)	00005400
000	HK(I) = -HK(1)	00005500
000	CK(I) = -CK(1)	00005600
000	AK(I) = -AK(1)	00005700
000	AFM(NEL,1) = AK(1)	00005800
000	AZM(NEL,1) = AZ(1)	00005900
000	100 CONTINUE	00006000
000	AO = AK(3)*AZ(2) - AK(4)*AZ(1)	00006100
000	BO = HK(2)*PZ(4) - HK(3)*PZ(1)	00006200
000	CO = (K(3)*CZ(1) - CK(4)*CZ(2)	00006300
000	AOJ(NEL) = AO	00006400
000	L	00006500
000	EO 200 M = 1,4	00006600
000	FO 150 N = 1,4	00006700
000	CALL GAUSS(1,AA)	00006800
000	TYPEA(M,N) = AA	00006900
000	CALL GAUSS(2,AA)	00007000
000	TYPEB(M,N) = AA	00007100
000	CALL GAUSS(3,AA)	00007200
000	TYPEC(M,N) = AA	00007300
000	CALL GAUSS(4,AA)	00007400
000	TYPEU(M,N) = AA	00007500
000	150 CONTINUE	00007600
000	CALL GAUSS(5,AA)	00007700
000	ET(NEL,M,1) = AA	00007800
000	ET(NEL,M+4,1) = AA / 2.	00007900
000	CALL GAUSS(6,AA)	00008000
000	ET(NEL,M+4,2) = AA	00008100
000	ET(NEL,M,3) = AA / 2.	00008200
000	200 CONTINUE	00008300
000	L	00008400
000	WRITE(2) TYPEA,TYPEB,TYPEC,TYPEU	00008500
000	L	00008600
000	L11 = L(1,1)	00008700
000	EO 300 I = 1,4	00008800
000	FO 300 J = 1,4	00008900

000	C		00009000
000		STIFF(I,J) = DP(NEL,1,1)*TYPEA(I,J)+DP(NEL,3,3)*TYPEC(I,J)	00009100
000		STIFF(I,J+4) = DP(NEL,1,2)*TYPEB(I,J)+DP(NEL,3,3)*TYPEB(J,I)	00009200
000		STIFF(J+4,I) = STIFF(I,J+4)	00009300
000		STIFF(I+4,J+4) = LP(NEL,2,2)*TYPEC(I,J)+DP(NEL,3,3)*TYPEA(I,J)	00009400
000	C		00009500
000		CM(I,J) = DFRS * TYPED(I,J)	00009600
000		CM(I+4,J+4) = CM(I,J)	00009700
000	C		00009800
000		VE(I,J) = ATD(1,1)*TYPEA(I,J) + ATD(3,3)*TYPEC(I,J)	00009900
000		VE(I,J+4) = ATD(3,3)*TYPEB(J,I)	00010000
000		VE(I+4,J) = ATD(3,3)*TYPEB(I,J)	00010100
000		VE(I+4,J+4) = ATD(2,2)*TYPEC(I,J) + ATD(3,3)*TYPEA(I,J)	00010200
000	C		00010300
000		300 CONTINUE	00010400
000	C		00010500
000		DO 110 I = 1,8	00010600
000		I1 = FK(NEL,I)	00010700
000		DO 110 J = 1,8	00010800
000		JJ = FK(NEL,J)	00010900
000		IF(I1.EQ.0.OR.JJ.EQ.0) GO TO 110	00011000
000		IF(I1.LT.JJ) GO TO 110	00011100
000		IF(I1.GT.IHBI) GO TO 104	00011200
000		L = JJ + (I1-J) * I1 / 2	00011300
000		GO TO 105	00011400
000		104 L = JJ + I1 + (I1-IHB) * IHB	00011500
000		105 XKL(L) = XKL(L) + STIFF(I,J)	00011600
000		XM(L) = XM(L) + CM(I,J)	00011700
000		CHAR(L) = CHAR(L) + VE(I,J)	00011800
000		110 CONTINUE	00011900
000	C		00012000
000		900 CONTINUE	00012100
000	C		00012200
000	C		00012300
000		666 FORMAT(8E15.7)	00012400
000		600 FORMAT(/' SUB STIFF1' /)	00012500
000		610 FORMAT(4F16.6)	00012600
000		RETURN	00012700
000		END	00012800
000		DELTA,SIH NASA*IPFS,DISPL,,163543132410	
000		SUBROUTINE DISPL(NI)	00000100
000		PARAMETER NFI=150,NFIS=150,MX= 5000,NF=NFI*2	00000200
000		COMMON /BLK0/ TITLE(20),INODE,NELEM,NAPC,NHC,MACT,ISW(5)	00000300
000		COMMON /BLK2/ ID(NF,2),IJKL(NELS,4),R(NFI),Z(NFI),KK(NELS,8)	00000400
000		COMMON /BLK3/ XKL(MX),P(NF),IMAX,IHB,IHBI,LT,LAST,NFREE	00000500
000	C		00000600
000		WRITE(6,620) MACT,NI	00000700
000		620 FORMAT(//' FORCE VECTOR SIZE =',15,' NI =',14/)	00000800
000		NI = LAST + 1	00000900
000		NI = LAST + NFRE	00001000
000		WRITE(6,667) (XKL(I),I=MM,NN)	00001100
000		667 FORMAT(9E14.6)	00001200
000	C		00001300
000		CALL FACIOR(XKL,IHB,IHBI,LT,LAST,NFREE)	00001400
000	C		00001500
000		CALL SOLIN (XKL,IHB,IHBI,LT,LAST,NFREE)	00001600
000	C		00001700

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000      WRITE(6,600)                                00001800
000      DO 280 I = 1,NBC                               00001900
000      J = (I-1)*2 + 1 + LAST                         00002000
000      JJ = J + 1                                       00002100
000      280 WRITE(6,610) I,XKL(J),XKL(JJ)             00002200
000      600 FORMAT(/// 10X,'GLOBAL DISPLACEMENT'//5X,'NODE',  00002300
000      110X,'Z-DISPL',10X,'P-DISPL')                00002400
000      610 FORMAT(5X,14,2E20.7)                      00002500
000      RETURN                                           00002600
000      END                                              00002700
000  DELT,SIH NASA*IPF9,SIRAIN,,226151130610
000      SUBROUTINE SIRAIN(NT)                            00000100
000      PARAMETER NF=150,NELS=150,MX= 5000,NF=NFT*2    00000200
000      COMMON /BLK0/ TITLE(20),INODE,NELEM,NAPC,NBC,MACT,ISW(5) 00000300
000      COMMON /BLK1/ W(4),F(4),AR(4),RR(4),CR(4),AZ(4),BZ(4),CZ(4), 00000400
000      * HQ(4),CN(4),DN(4),TYPEA(4,4),TYPEB(4,4),TYPEC(4,4),TYPEU(4,4), 00000500
000      * AU,BU,CU,IC,JC,KC,LC,NFL                     00000600
000      COMMON /BLK2/ ID(NF,2),IJKL(NELS,4),R(NFT),7(NFT),KK(NELS,8) 00000700
000      COMMON /BLK3/ XKL(MX),P(NF),IMAX,IHP,IHRI,LT,LAST,NFREE 00000800
000      COMMON /BLK4/ DE(3,3),AII(3,3),ADI(3,3),BD(3,3),STIFF(8,8),CM(8,8), 00000900
000      * VE(8,8),AI(NELS) \                             00001000
000      COMMON /BLK5/ GR(NELS,3),STRAIR(NELS,3),ALPHA,BETA,GAMMA,DELT,XX, 00001100
000      * A1,XNU,E,SM,SMS,VOIDI,CAPA,HAMD,BET          00001200
000      COMMON /BLK6/ BT(NELS,8,3),ARM(NELS,4),AZM(NELS,4),AOJ(NELS) 00001300
000      COMMON /BLK8/ DP(NELS,3,3),PRINS(NELS,3)        00001400
000      DIMENSION U(4),V(4),U(3,3)                    00001500
000      C                                                00001600
000      WRITE(6,600) N1                                  00001700
000      C                                                00001800
000      DO 900 NEL = 1,NELEM                             00001900
000      DO 200 I = 1,4                                    00002000
000      IN = IJKL(NEL,1)                                  00002100
000      II = (IN-1) * 2 + 1 + LAST                       00002200
000      V(1) = XKL(II)                                    00002300
000      200 U(1) = XKL(II+1)                              00002400
000      620 FORMAT(4E15.6)                               00002500
000      C                                                00002600
000      DO 210 J = 1,3                                    00002700
000      DO 210 J = 1,3                                    00002800
000      210 D(1,J) = DP(NEL,1,J)                        00002900
000      C                                                00003000
000      C        COMPUTE STRESSES AT THE CTR.            00003100
000      C                                                00003200
000      EZ = 0.                                           00003300
000      ER = 0.                                           00003400
000      GR = 0.                                           00003500
000      DO 100 I = 1,4                                    00003600
000      EZ = EZ - ARM(NEL,I) * V(I) / AOJ(NEL)          00003700
000      ER = ER - ARM(NEL,I) * U(I) / AOJ(NEL)          00003800
000      100 GR = GR - (ARM(NEL,I) * U(1) + AZM(NEL,I) * V(I)) / AOJ(NEL) 00003900
000      SIGMZ = U(1,1) * EZ + U(1,2) * ER              00004000
000      SIGMR = U(2,2) * ER + U(2,1) * EZ              00004100
000      SHEAR = U(3,3) * GR                             00004200
000      650 FORMAT(15,15.6)                             00004300
000      C                                                00004400
000      GR(NEL,1) = SIGMZ + GR(NEL,1)                  00004500
000      GR(NEL,2) = SIGMR + GR(NEL,2)                  00004600

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000      GP(NEL,3) = SHEAR + GB(NEL,3)      00004700
000      STRAIN(NEL,1) = SIGM2      00004800
000      STRAIN(NEL,2) = SIGM2      00004900
000      STRAIN(NEL,3) = SHEAR      00005000
000      C      00005100
000      C      COMPUTE PRINCIPAL STRESS. (COMP. IS POSITIVE HERE)      00005200
000      C      00005300
000      SG1 = ((GB(NEL,1) - GB(NEL,2)) / 2.) **2 + GP(NEL,3)**2)**.5      00005400
000      SG1 = (GB(NEL,1) + GB(NEL,2)) / 2.      00005500
000      PRINS(NEL,1) = SG1 + SGH      00005600
000      PRINS(NEL,2) = SG1 - SGH      00005700
000      PRINS(NEL,3) = SGH      00005800
000      000 CONTINUE      00005900
000      C      00006000
000      WRITE(6,010) (1,(GB(I,J),J=1,3),(PRINS(I,K),K=1,3),I=1,NELEM)      00006100
000      000 FORMAT(1H1,10X,'LOCAL STRESS AT THE END OF',14,' LOAD INCREMENT',/00006200
000      1 5X,'ELEM',10X,'STRESS AT CTR',30X,'PRINCIPAL STRESS'/)      00006300
000      610 FORMAT(10,3F13.5,3X,3E13.5)      00006400
000      RETURN      00006500
000      END      00006600
000      MFLI,SIH,NASA*IPF9,PLASTIC,,,163544132410      00006700
000      SUBROUTINE PLASTC      00006800
000      PARAMETER NF=150,NFLE=150,MX= 500,NF=NFT*2      00006900
000      COMMON /BLK0/ IJILL(20),INODE,NFLEM,NAPC,NPC,MACT,IS#(5)      00007000
000      COMMON /BLK2/ IF(NF,2),IUKL(NFIS,4),R(NF1),Z(NFT),KK(NELS,8)      00007100
000      COMMON /BLK3/ XKL(MX),P(NF),IMAX,IHF,IHF1,LT,LAST,NFREL      00007200
000      COMMON /BLK4/ U(3,3),AIF(3,3),AP(3,3),BD(3,3),STIFF(8,8),CM(8,8),      00007300
000      * VE(8,8),A1(NFLE)      00007400
000      COMMON /BLK7/ UDB(NF),UDBR(NF),FOV(NF),HLD(NF),DB(NF),DISP(NF),      00007500
000      * DOUT(NF)      00007600
000      COMMON /DYN/ CBAR(MX),XN(MX),AA(MX)      00007700
000      C      00007800
000      CALL ZERO(AA,MX,1)      00007900
000      NLOAD = -IS#(1)      00008000
000      SCAL = NLOAD      00008100
000      DO 100 I = 1,MAC1      00008200
000      P(1) = P(1) / SCAL      00008300
000      100 XKL(LAST+I) = P(1)      00008400
000      C      00008500
000      DO 900 N1 = 1,NLOAD      00008600
000      C      00008700
000      IF(N1.EQ.1) GO TO 000      00008800
000      CALL DIAGNL(XPL,P,NFREL,IHB,IHF1,LT,LAST)      00008900
000      C      00009000
000      CALL STIFF2      00009100
000      C      00009200
000      000 CALL DISPL(P1)      00009300
000      DO 120 I = 1,NFREL      00009400
000      AA(1) = AA(1) + XKL(LAST+I)      00009500
000      120 DISP(I) = AA(1)      00009600
000      C      00009700
000      WRITE(6,000) N1      00009800
000      DO 130 I = 1,INODE      00009900
000      JU = I + 2      00010000
000      II = JU - 1      00010100
000      130 WRITE(6,010) I,AA(IJ),AA(JJ)      00010200
000      C      00010300

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000      CALL STRAIN(NI)                                000003700
000      C                                                000003800
000      900 CONTINUE                                    000003900
000      C                                                000004000
000      600 FORMAT(1H1,5X,'TOT. DISPL. AT THE END OF',I4,' LOAD INCREMENT',// 000004100
000      1 5X,'1-DISPL',5X,'2-DISPL',15X,'R-DISPL'//) 000004200
000      610 FORMAT(1H0,2E20.7)                        000004300
000      STOP                                            000004400
000      END                                             000004500
000      WELL,SIH NASA*1PF%.STIFF2,,163551132410
000      SUBROUTINE STIFF2                                000000100
000      PARAMETER NFI=150,NELS=150,MX= 5000,NF=NFI*2 000000200
000      COMMON /BLK0/ IITLE(20),INONE,NFLEM,NAPC,NHC,MACT,ISW(5) 000000300
000      COMMON /BLK1/ W(4),H(4),AR(4),PR(4),CR(4),A7(4),B7(4),CZ(4), 000000400
000      * BN(4),CN(4),DN(4),TYPEA(4,4),TYPEB(4,4),TYPEC(4,4),TYPEU(4,4), 000000500
000      * AO,BO,CO,IC,JC,KC,LC,NEL 000000600
000      COMMON /BLK2/ ID(NF,2),IJKL(NELS,4),R(NFI),Z(NFI),KK(NELS,8) 000000700
000      COMMON /BLK3/ XKL(MY),P(NF),IMAX,IHR,IHRI,LT,LAST,NFREE 000000800
000      COMMON /BLK4/ U(3,3),AIP(3,3),AD(3,3),BD(3,3),STIFF(8,8),CM(8,8), 000000900
000      * VE(8,8),A1(NELS) 000001000
000      COMMON /BLK5/ GR(NELS,3),STRAIB(NELS,3),ALPHA,BETA,GAMMA,DELT,XK, 000001100
000      * AT,XIPU,E,SM,SMS,CUTLI,CAPA,RAND,BET 000001200
000      COMMON /BLK6/ LEL(3,3),PRINS(NELS,3) 000001300
000      C                                                000001400
000      REWIND 2                                         000001500
000      C                                                000001600
000      XNUS = XNU * 1.0                                000001700
000      VA = YNU + 1.0                                  000001800
000      VB = 1.+XNUS-1.0                                000001900
000      VC = 2.*(XNUS-1.0) - 1.0                        000002000
000      C                                                000002100
000      DO 900 NEL = 1,NFLEM                            000002200
000      READ(2) TYPEA,TYPEB,TYPEC,TYPEU                000002300
000      IF(NEL.LE.ISW(5)) GO TO 800                    000002400
000      DO 200 I = 1,3                                  000002500
000      DO 200 J = 1,3                                  000002600
000      200 DP(NEL,I,J) = U(1,J)                      000002700
000      C                                                000002800
000      SIGZ = QB(NEL,1) + STRAIB(NEL,1)/2.            000002900
000      SIGR = QB(NEL,2) + STRAIB(NEL,2)/2.            000003000
000      TAUZ = QB(NEL,3) + STRAIB(NEL,3)/2.            000003100
000      DELP = VA * (STRAIB(NEL,1)+STRAIB(NEL,2))/3.   000003200
000      PP = VA * (SIGZ+SIGR) / 3.                     000003300
000      PSU = PP * PP                                    000003400
000      SZZ = 2.*VB*SIGZ + VC*SIGR                    000003500
000      SRK = 2.*VB*SIGR + VC*SIGZ                    000003600
000      SZR = 6.*TAUZ                                    000003700
000      TJ = (VB*SIGZ*SIGZ+VB*SIGR*SIGR+VC*SIGZ*SIGR)/3. + TAUZ*TAUZ 000003800
000      CALL AREAA(NEL,AREA)                            000003900
000      RATIO = AREA / A1(NEL)                         000004000
000      VOIDR = RATIO * (1.+VOIDI) - 1.                 000004100
000      VDR = VOIDR + 1.                                000004200
000      TTJ = 3.*TJ                                      000004300
000      ETM = 1.+TTJ/(PSU+SMS)                          000004400
000      POW = 1.-CAPA/RAND                              000004500
000      PU = PP*ETM*POW                                  000004600
000      AA = SMS * (2.*PP-PO) / 3.                    000004700

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000	B6 = 3.*AA*VOR*SMS*PP*PO / BET	00004800
000	AR(1) = SZZ + AA	00004900
000	AR(2) = SRR + AA	00005000
000	AR(3) = SZR	00005100
000	HR(1) = SZZ	00005200
000	HR(2) = SRR	00005300
000	HR(3) = SZR	00005400
000	DF1 = 0.	00005500
000	DFK = 0.	00005600
000	DO 220 I = 1,3	00005700
000	DF1 = DF1 + AR(I) * STRAIB(NEL,I)	00005800
000	220 DFK = DFK + BR(I) * STRAIB(NEL,I)	00005900
000	DFJ = PO * SMS * DELP	00006000
000	POW = -CAPA/RAND	00006100
000	DFK = ETM **POW * (DFK-2.*TTJ/PP*DELP)	00006200
000	DF = DFI - DFJ - DFK * (1.-CAPA/RAND)	00006300
000	ASQ = SMS * PP * (PO-PP)	00006400
000	ETA = SQRT(TTJ) / PP	00006500
000	WRITE(6,620) VOIDR,TTJ,ASQ,DF,NEL,SM,ETA	00006600
000	620 FORMAT(5X,'VOIDR=',F10.5,' 3J=',E12.5,' ASQ=',E12.5,' DF=',	00006700
000	1 E12.5,' 17,' M=',E12.5,' ETA=',E12.5)	00006800
000	IF(DK.LT.0.) GO TO 800	00006900
000	DO 300 I = 1,3	00007000
000	HR(I) = 0.	00007100
000	CR(I) = 0.	00007200
000	DO 300 J = 1,3	00007300
000	BR(I) = BR(I) + D(I,J)*AR(J)	00007400
000	300 CR(I) = CR(I) + AR(J)*D(J,I)	00007500
000	DEN = 0.	00007600
000	DO 310 I = 1,3	00007700
000	310 DEN = DEN + AR(I)*BR(I)	00007800
000	DEN = DEN + BR	00007900
000	DO 320 I = 1,3	00008000
000	DO 320 J = 1,3	00008100
000	320 DP(NEL,I,J) = D(I,J) - BR(I)*CR(J) / DEN.	00008200
000	C	00008300
000	800 CONTINUE	00008400
000	DO 100 I = 1,4	00008500
000	DO 100 J = 1,4	00008600
000	STIFF(I,J) = DP(NEL,1,1)*TYPEA(I,J)+DP(NEL,3,3)*TYPEC(I,J)	00008700
000	STIFF(I,J+4) = DP(NEL,1,2)*TYPER(I,J)+DP(NEL,3,3)*TYPEB(J,I)	00008800
000	STIFF(J+4,1) = STIFF(I,J+4)	00008900
000	100 STIFF(I+4,J+4) = DP(NEL,2,2)*TYPEC(I,J)+DP(NEL,3,3)*TYPEA(I,J)	00009000
000	C	00009100
000	DO 210 I = 1,8	00009200
000	II = KK(NEL,I)	00009300
000	DO 210 J = 1,8	00009400
000	JJ = KK(NEL,J)	00009500
000	IF(II.EQ.0.OR.JJ.EQ.0) GO TO 210	00009600
000	IF(II.LT.JJ) GO TO 210	00009700
000	IF(II.GT.IHBI) GO TO 214	00009800
000	L = JJ + (II-1) * II / 2	00009900
000	GO TO 215	00010000
000	214 L = JJ + LT + (II-IHB) * IHBI	00010100
000	215 XKL(L) = XKL(L) + STIFF(I,J)	00010200
000	210 CONTINUE	00010300
000	900 CONTINUE	00010400

000	C		00010500
000		RETURN	00010600
000		END	00010700
000		DELTA,SIH,NASA*IPF,STFP,,,163560132410	
000		SUBROUTINE STEP	00000100
000		PARAMETER NFI=150,NELS=150,MX= 5000,NF=NFI*2	00000200
000		COMMON /BLK1/ TITLE(20),INODE,NELEM,NAPC,NHC,MACT,ISW(5)	00000300
000		COMMON /BLK2/ XKL(MX),P(NF),IMAX,IHB,IHB1,LT,LAST,NFREE	00000400
000		COMMON /BLK3/ GH(NELS,3),STRAIB(NELS,3),ALPHA,BETA,GAMMA,DELT,XK,	00000500
000		* AI,XMU,E,SI,SPS,VOIDI,CAPA,RAMD,BET	00000600
000		COMMON /BLK7/ UDB(NF),UDD(NF),FOV(NF),UDD(NF),DB(NF),DISP(NF),	00000700
000		* DDOT(NF)	00000800
000		COMMON /BLK8/ UP(NELS,3,3),SIGB(NELS,3)	00000900
000		COMMON /DYN/ CBAR(MX),XM(MX),AA(MX)	00001000
000		NTIME=1	00001100
000		NDELT = ISW(1)	00001200
000		N = MACT	00001300
000		NPRNT = ISW(5)	00001400
000		IKOUNT = 250	00001500
000		KOUNT = IKOUNT	00001600
000		IPLT = 1	00001700
000		CALL ZERO(SIGB,NELS,3)	00001800
000	C		00001900
000		DO 2 I = 1, LAST	00002000
000	2	AA(I) = XM(I) + CBAR(I) * DELT / 2. + XKL(I) * DELT * DELT / 4.	00002100
000		DO 150 I = 1, NFREE	00002200
000	150	AA(I+LAST) = P(I)	00002300
000	C		00002400
000		CALL FACTOR(AA,IHB,IHB1,LT,LAST,NFREE)	00002500
000		CALL SOLTN (AA,IHB,IHB1,LT,LAST,NFREE)	00002600
000	C		00002700
000		DO 3 I=1,N	00002800
000		UDB(I)=0.	00002900
000		UDDR(I)=0.	00003000
000		FOV(I)=0.	00003100
000		UDD(I) = AA(LAST+1)	00003200
000	3	DB(I)=0.	00003300
000		DO 4 I=1,N	00003400
000		DISP(I)= DB(I) + DELT*UDB(I) +(UDD(I) +UDD(I))*DELT**2/4.	00003500
000	4	DDOT(I)= UDB(I) + (UDD(I) +UDD(I))*DELT /2.	00003600
000		WRITE(6,100) NTIME	00003700
000	100	FORMAT (25X,'DISPLACEMENTS FOR TIME INCREMENT',I5,5X,'LINEAR')	00003800
000		WRITE (6,6997) (DISP(I),I=1,N)	00003900
000	6997	FORMAT(8E15.6)	00004000
000	101	FORMAT(8E15.6)	00004100
000		WRITE(6,102)	00004200
000	102	FORMAT('VELOCITY')	00004300
000		WRITE(6,101) (DDOT(I),I=1,N)	00004400
000	C		00004500
000	C	INITIALIZE DB,STRAIB	00004600
000	C		00004700
000		CALL ZERO(STRAIB,NELS,3)	00004800
000		CALL ZERO(GH,NELS,3)	00004900
000	C		00005000
000		CALL STAN(NTIME)	00005100
000	C		00005200
000	C	NOW READY FOR NEW INCREMENTS..	00005300

000	C		00005400
000	10	CONTINUE	00005500
000	C		00005600
000		JJ = IHB1	00005700
000		DO 200 I = 1,NFREE	00005800
000		JJ = JJ + 1	00005900
000		SUN = 0.	00006000
000		SUM = 0.	00006100
000		IF(I.GE.IHB) GO TO 180	00006200
000		L = (I+1)*I / 2 - 1	00006300
000		DO 170 J = 1,JJ	00006400
000		IF(J.LE.1) L = L + 1	00006500
000		IF(J.GT.1.AND.J.LE.IHB) L = L + J - 1	00006600
000		IF(J.GT.IHB) L = L + IHB1	00006700
000		SUN = SUN + XM(L)*UB(J)	00006800
000	170	SUM = SUM + CBAR(L)*(UB(J)+UDDR(J)*DELT/2.) + XKL(L)*(DB(J)+DELT*	00006900
000		1UBB(J)+UDDDB(J)*DELT*DELT/4.)	00007000
000		GO TO 199	00007100
000	180	II = I - IHB + 1	00007200
000		L = LT + (II-1) * IHB	00007300
000		IF(JJ.GT.NFREE) JJ = NFREE	00007400
000		DO 190 J = II,JJ	00007500
000		IF(J.LE.1) L = L + 1	00007600
000		IF(J.GT.1) L = L + IHB1	00007700
000		SUN = SUN + XM(L)*UB(J)	00007800
000	190	SUM = SUM + CBAR(L)*(UB(J)+UDDR(J)*DELT/2.) + XKL(L)*(DB(J)+DELT*	00007900
000		1UBB(J)+UDDDB(J)*DELT*DELT/4.)	00008000
000	199	XM(LAST+1) = SUM	00008100
000		CHAR(LAST+1) = SUM	00008200
000	200	CONTINUE	00008300
000	C		00008400
000		IF(NTIME.GT.NDELT) GO TO 113	00008500
000		DO 12 I = 1,NFREE	00008600
000	12	AA(I+LAST) = P(1) - FOV(I) - XM(LAST+I) - CBAR(LAST+I)	00008700
000	C		00008800
000	300	CALL SOLTN (AA,IHB,IHB1,LT,LAST,NFREE)	00008900
000	C		00009000
000		DO 13 I=1,N	00009100
000		FOV(I)=0.	00009200
000		UDD(I) = AA(LAST+1)	00009300
000		DISP(I)= DB(1) + DELT*UBB(1) + (UDD(1) + UDD(1))*DELT**2/4.	00009400
000	13	DDOT(I)= UDB(I) + (UDD(1) + UDD(1))*DELT /2.	00009500
000	C		00009600
000		CALL STAN(NTIME)	00009700
000	C		00009800
000		IF(NTIME.NE.NPKNT) GO TO 310	00009900
000		NPKNT = NPKNT + ISW(3)	00010000
000		WRITE(6,105) NTIME	00010100
000	105	FORMAT(10X,'DISPLACEMENT AT NTIME =',I5/5X,'NODE NO	Z-DISPL 00010200
000	1	R-DISPL')	00010300
000		DO 106 I = 1,INODE	00010400
000		JJ = I * 2	00010500
000		II = JJ - 1	00010600
000	106	WRITE(6,620) I,DISP(II),DISP(JJ)	00010700
000	620	FORMAT(I9,2E15.6)	00010800
000		II = LAST + 1	00010900
000		JJ = LAST + NFREE	00011000

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000      WRITE(6,600)
000      WRITE(6,101) (AA(I),I=1I,JJ)
000      WRITE(6,610)
000      WRITE(6,101) (CPAK(I),I=1I,JJ)
000      600 FORMAT(/5X,'(0),FORCE'/)
000      610 FORMAT(/5X,'FP'/)
000      310 CONTINUE
000      GO TO 10
000
000      C
000      113 WRITE(6,1968)
000      1968 FORMAT(/2X,'END OF ANALYSIS'///)
000      RETURN
000      END
000      WELT,SIH NASA*1PF$,STAN,,,163570132410
000      SUBROUTINE STAN(NTIME)
000      PARAMETER NFI=150,NELS=150,MX= 5000,NF=NFI*2
000      COMMON /BLK0/ TITLE(20),INODE,NELEM,NAPC,NBC,MACT,ISW(5)
000      COMMON /BLK2/ IN(NF,2),IJKL(NELS,4),R(NFI),Z(NFI),KK(NELS,8)
000      COMMON /BLK3/ XKL(MX),P(NF),IMAX,IHR,IHRI,LT,LAST,NFRFE
000      COMMON /BLK4/ D(3,3),ATD(3,3),AD(3,3),BD(3,3),STIFF(8,8),CM(8,8),
000      * VE(8,8),A1(NELS)
000      COMMON /BLK5/ GP(NELS,3),STAIR(NELS,3),ALPHA,BETA,GAMMA,DELI,XK,
000      * AT,XMU,E,SM,SMS,VOIDI,CAPA,RAND,BET
000      COMMON /BLK6/ BT(NELS,8,3),ARM(NELS,4),AZM(NELS,4),AOJ(NELS)
000      COMMON /BLK7/ UDB(NF),UDDB(NF),FOV(NF),UDD(NF),DB(NF),DISP(NF),
000      * UDOT(NF)
000      COMMON /BLK8/ DP(NELS,3,3),SIGR(NELS,3)
000      COMMON /LYN/ CBAR(MX),XM(MX),AA(MX)
000      DIMENSION V(4),H(4),VD(4),EPS(3),EPSD(3),SIGE(3),SIGV(3),
000      * SIGF(3),SIGT(3),FV(8)
000
000      C
000      CALL ZERO(XM,MX,1)
000      REWIND 2
000      NN = NTIME / ISW(3)
000      NM = NN * ISW(3)
000      IRITE = 0
000      IF(NM.EQ.NTIME) IRITE = 1
000      IF(NM.EQ.NTIME) WRITE(6,600) NTIME
000      DO 900 NEL = 1,NELEM
000      DO 100 I = 1,4
000      IN = IJKL(NEL,I)
000      II = (IN-1)*2 + 1
000      V(1) = DISP(II)
000      U(1) = DISP(II+1)
000      VD(I) = UDOT(II)
000      UD(I) = UDOT(II+1)
000
000      100 CONTINUE
000      CALL ZERO(EPS,3,1)
000      CALL ZERO(EPSD,3,1)
000      DO 110 I = 1,4
000      EPS(1) = EPS(1) + ARM(NEL,I)*V(I) / AOJ(NEL)
000      EPS(2) = EPS(2) + AZM(NEL,I)*U(I) / AOJ(NEL)
000      EPS(3) = EPS(3)+(ARM(NEL,I)*U(I)+AZM(NEL,I)*V(I)) /AOJ(NEL)
000      EPSD(1) = EPSD(1) + ARM(NEL,I)*VD(I) / AOJ(NEL)
000      EPSD(2) = EPSD(2) + AZM(NEL,I)*UD(I) / AOJ(NEL)
000      110 EPSD(3) = EPSD(3)+(ARM(NEL,I)*UD(I)+AZM(NEL,I)*VD(I)) /AOJ(NEL)
000      DO 120 I = 1,3

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000	SIGE(I) = 0.	00004400
000	SIGV(I) = 0.	00004500
000	SIGF(I) = 0.	00004600
000	DO 119 J = 1,3	00004700
000	SIGE(I) = SIGE(I) + B(1,J)*EPS(J)	00004800
000	SIGV(I) = SIGV(I) + ATU(1,J)*EPSD(J)	00004900
000	119 SIGF(I) = SIGF(I) + BD(I,J)*STRAIB(NEL,J) + AD(I,J)*GB(NEL,J)	00005000
000	120 SIGT(I) = SIGE(I) + SIGV(I) + SIGF(I)	00005100
000	C	00005200
000	C COMPUTE PRINCIPAL STRESSES	00005300
000	C COMPRESSION IS POSITIVE HERE	00005400
000	C	00005500
000	SQR = (((SIGT(1)-SIGT(2))/2.))**2 + SIGT(3)**2) **.5	00005600
000	SGI = -(SIGT(1) + SIGT(2)) / 2.	00005700
000	EPS(1) = SGI + SQR	00005800
000	EPS(2) = SGI - SQR	00005900
000	EPS(3) = SQR	00006000
000	IF(MM.EQ.NTIME) WRITE(6,610) NEL,SIGE,SIGV,SIGT,EPS	00006100
000	C	00006200
000	IF(ISW(2).EQ.1) CALL PSTIFF(NEL,SIGE,IPLST,IRTF)	00006300
000	IF(ISW(2).EQ.0) GO TO 900	00006400
000	C	00006500
000	C COMPUTE VISC. FORCE FOR EACH ELEMENT	00006600
000	C	00006700
000	DO 130 I = 1,8	00006800
000	FV(I) = 0.	00006900
000	DO 130 J = 1,3	00007000
000	130 FV(I) = FV(I) + B(NEL,I,J) * SIGF(J)	00007100
000	C NOW ASSEMBLE IN GLOBAL FORM	00007200
000	DO 140 I = 1,8	00007300
000	II = KK(NEL,I)	00007400
000	IF(II.EQ.0) GO TO 140	00007500
000	F0V(II) = F0V(II) + FV(I)	00007600
000	IF(ISW(2).NE.1.OR.IPLST.NE.1) GO TO 140	00007700
000	DO 139 J = 1,8	00007800
000	JJ = KK(NEL,J)	00007900
000	IF(JJ.EQ.0) GO TO 139	00008000
000	IF(II.LT.JJ) GO TO 139	00008100
000	IF(II.GT.IHBI) GO TO 137	00008200
000	L = JJ+(II-1)*IJ/2	00008300
000	GO TO 138	00008400
000	137 L = JJ + LI + (II-IHB)*IHBI	00008500
000	138 XM(L) = XM(L) + STIFF(1,J)	00008600
000	139 CONTINUE	00008700
000	140 CONTINUE	00008800
000	C	00008900
000	C UPDATE (GB) AND (STRAIB). (ATRAIN RATE)	00009000
000	C	00009100
000	DO 150 I = 1,3	00009200
000	SIGR(NEL,I) = -SIGE(I)	00009300
000	GR(NEL,I) = ALPHA*GR(NEL,I)+BETA*STRAIB(NEL,I)+GAMMA*EPSD(I)	00009400
000	150 STRAIB(NEL,I) = EPSD(I)	00009500
000	C	00009600
000	900 CONTINUE	00009700
000	C	00009800
000	C UPDATE DISPL.,VEL., AND ACCEL.	00009900
000	C	00010000

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000      160 NTIME = NTIME + 1
000      DO 910 I = 1,MACT
000          DF(I) = DISP(I)
000          UDB(I) = DUOT(I)
000      910 UDB(I) = UDB(I)
000      600 FORMAT(///10X,'NTIME =',I5/2X,'ELEM',T10,'ELASTIC STRESS',T40,
000          1,'VISCIOUS STRESS',T70,'101.STRESS',T100,'PRINCIPAL STRESS'/)
000      610 FORMAT(I5,12E10.3)
000      C
000      620 FORMAT(///10X,'PRINCIPAL STRESS. NTIME =',I5/)
000      630 FORMAT(I10,3E13.5)
000      C
000      RETURN
000      END
000      WELT,SIH NASA*TPF$,PSTIFF,,,163602132410
000      SUBROUTINE PSTIFF(NEL,SIGT,IPLST,IRITE)
000      PARAMETER NFT=150,NELS=150,MX= 5000,NF=NFT*2
000      COMMON /BLK0/ TITLE(20),INODE,NELEM,NAPC,NBC,MACT,ISW(5)
000      COMMON /BLK1/ W(4),H(4),AR(4),BR(4),CR(4),AZ(4),BZ(4),CZ(4),
000      * FN(4),CN(4),DN(4),TYPEA(4,4),TYPEB(4,4),TYPEC(4,4),TYPED(4,4),
000      * AU,BU,CU,IV,JU,KU,LC,MEL
000      COMMON /BL2/ D(3,3),AD(3,3),BD(3,3),STIFF(8,8),CM(8,8),
000      * VE(6,6),ALF(NELS)
000      COMMON /BLK3/ UP(NELS,3),STRAIR(NELS,3),ALPHA,BETA,GAMMA,DELTA,XK,
000      * AT,XNU,L,SM,SMS,VOIDI,CAPA,RAMD,BET
000      COMMON /BLK4/ UP(NELS,3,3),SIGB(NELS,3)
000      DIMENSION SIGT(3)
000      C
000      C      COMPRESSION IS POSITIVE
000      C
000      CALL ZERO(STIFF,8,8)
000      READ(2) (TYPEA,TYPEB,TYPEC,TYPED
000      IPLST = -1
000      IF(NEL.LE.ISW(5)) GO TO 800
000      XNUS = XNU * XNU
000      VA = XNU + 1.
000      VB = 1.+XNUS-XNU
000      VC = 2.*(XNUS-XNU) - 1.
000      DSIGZ = -SIGT(1)-SIGB(NEL,1)
000      DSIGR = -SIGT(2)-SIGB(NEL,2)
000      DTAUZ = -SIGT(3)-SIGB(NEL,3)
000      SIGZ = -SIGT(1) + DSIGZ/2.
000      SIGR = -SIGT(2) + DSIGR/2.
000      TAUZ = -SIGT(3) + DTAUZ/2.
000      DELP = VA*(DSIGZ+DSIGR) / 3.
000      PP = VA * (SIGZ+SIGR) / 3.
000      PSQ = PP * PP
000      SZZ = 2.*VB*SIGZ + VC*SIGR
000      SRK = 2.*VB*SIGR + VC*SIGZ
000      SZK = 6.*TAUZ
000      TJ = (VB*SIGZ*SIGZ+VB*SIGR*SIGR+VC*SIGZ*SIGR)/3. + TAUZ*TAUZ
000      TTJ = 3.*TJ
000      FTM = 1.+TTJ/(PSQ*SMS)
000      POW = 1.-CAPA/RAMD
000      PC = PP*ETM**POW
000      CALL AREA(NEL,AREA)
000      RATIO = AREA / A1(NEL)

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000      VOIDK = RATIO * (1.+VOIDI) - 1.      000004300
000      VK = VOIDK + 1.      000004400
000      AA = SMS * (2.*PP-PO) / 3.      000004500
000      PB = 3.*AA*VDR*SMS*PP*PO / RET      000004600
000      AK(1) = SZZ + AA      000004700
000      AK(2) = SRK + AA      000004800
000      AK(3) = SZK      000004900
000      PR(1) = SZZ      000005000
000      PR(2) = SRK      000005100
000      PR(3) = SZK      000005200
000      DF1 = 0.      000005300
000      LFK = 0.      000005400
000      DO 220 I = 1,3      000005500
000      DF1 = DF1 + AR(I) * STRAIB(NEL,I)      000005600
000      220 LFK = LFK + PR(I) * STRAIB(NEL,I)      000005700
000      IFJ = PO * SMS * DELP      000005800
000      PGV = -CAPA/RAND      000005900
000      LFK = ETM **POW * (LFK-2.*ITJ/PP*DELP)      000006000
000      DF = DF1 - IFJ - LFK * (1.-CAPA/RAND)      000006100
000      ASG = SMS * PP * (PO-PP)      000006200
000      ETA = SQRT(ITJ) / PP      000006300
000      IF(DF.LT.0.) GO TO 800      000006400
000      IPLSI = 1      000006500
000      IF(IRITE.EQ.1) WRITE(6,620) VOIDK,ITJ,ASG,DF,NEL,SM,ETA      000006600
000      620 FORMAT(5X,'VOIDR=',F10.5,' 3J=',E12.5,' ASG=',E12.5,' DF=',      000006700
000      1 E12.5,' 17,' M=',E12.5,' FIA=',E12.5)      000006800
000      DO 300 I = 1,3      000006900
000      PR(I) = 0.      000007000
000      CR(I) = 0.      000007100
000      DO 300 J = 1,3      000007200
000      BR(I) = BR(I) + D(1,J)*AK(J)      000007300
000      300 CR(I) = CR(I) + AK(J)*D(J,1)      000007400
000      DEN = 0.      000007500
000      DO 310 I = 1,3      000007600
000      310 DEN = DEN + AR(I)*BR(I)      000007700
000      DEN = DEN + BR      000007800
000      DO 320 I = 1,3      000007900
000      DO 320 J = 1,3      000008000
000      320 DP(NEL,I,J) = -BR(1) * CR(J) / DEN      000008100
000      600 FORMAT(15,' AREA=', VOIDR, PO, UPU,,,'4E15.6)      000008200
000      610 FORMAT(3E15.6)      000008300
000      C      000008400
000      DO 100 I = 1,4      000008500
000      DO 100 J = 1,4      000008600
000      STIFF(I,J) = DP(NEL,1,1)*TYPEA(I,J)+DP(NEL,3,3)*TYPEC(I,J)      000008700
000      STIFF(I,J+4) = DP(NEL,1,2)*TYPER(I,J)+DP(NEL,3,3)*TYPEB(J,I)      000008800
000      STIFF(J+4,I) = STIFF(I,J+4)      000008900
000      100 STIFF(I+4,J+4) = DP(NEL,2,2)*TYPEC(I,J)+DP(NEL,3,3)*TYPEA(I,J)      000009000
000      C      000009100
000      800 RETURN      000009200
000      END      000009300
000      WELT,SIH NASA*TYPE$.FACTOR,,226224130610
000      SUBROUTINE FACTOR(XK,IHB,IHRI,LT,LAST,NFREE)      000000100
000      C      THIS SUBROUTINE PERFORMS FACTORING      000000200
000      DIMENSION XK(1)      000000300
000      N = NFREE      000000400
000      IHB1 = IHRI      000000500

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000	DO 4 I=1,N	00000600
000	IF(I.GT.IHB1) GO TO 2	00000700
000	K=1	00000800
000	M=K+(I-1)*1/2	00000900
000	GO TO 3	00001000
000	2 K=I-IHB1	00001100
000	M=K+LT+(I-IHB)*IHB1	00001200
000	3 J=I+IHB1	00001300
000	IF(J.GT.N) GO TO 4	00001400
000	JJ=I+IHB1	00001500
000	GO TO 5	00001600
000	4 JJ=N	00001700
000	5 H=0.0	00001800
000	LA=1-1	00001900
000	LI=I+1	00002000
000	IF(LA.EQ.0) GO TO 6	00002100
000	DO 7 L=K,LA	00002200
000	IF(L.GT.IHB1) GO TO 50	00002300
000	J = (L+1)*L/2	00002400
000	GO TO 51	00002500
000	50 J = LT + IHB*(L-IHB1)	00002600
000	51 A = XK(M)	00002700
000	H = H+A*A*XK(J)	00002800
000	7 M=M+1	00002900
000	6 A=XK(M)	00003000
000	XK(M)=A-B	00003100
000	IF(I.FQ.N) GO TO 8	00003200
000	DO 9 J=LB,JJ	00003300
000	SUM=0.0	00003400
000	IF(J.GT.IHB1) GO TO 10	00003500
000	K=1	00003600
000	MM=K+(J-1)*J/2	00003700
000	GO TO 11	00003800
000	10 K=J-IHB1	00003900
000	MM=K+LT+(J-IHB)*IHB1	00004000
000	11 IF(LA.EQ.0) GO TO 9	00004100
000	IF(K.GT.LA) GO TO 9	00004200
000	DO 12 JA=K,LA	00004300
000	L=M-1+JA	00004400
000	IF(JA.GT.IHB1) GO TO 13	00004500
000	L1=(JA+1)*JA/2	00004600
000	GO TO 14	00004700
000	13 L1=LI+IHB*(JA-IHB1)	00004800
000	14 SUM=SUM+XK(MM)*XK(L)*XK(L1)	00004900
000	12 MM=MM+1	00005000
000	9 XK(MM)=(XK(MM)-SUM)/XK(M)	00005100
000	8 CONTINUE	00005200
000	RETURN	00005300
000	END	00005400
000	DELTA,SIH NASA*IPF*,SOLTN,,,226226130610	
000	SUBROUTINE SOLTN(XK,IHB,IHB1,LT,LAST,NFREE)	00000100
000	C THIS PORTION OF SUBROUTINE PERFORMS FORWARD-SUBSTITUTION	00000200
000	DIMENSION XK(1)	00000300
000	C	00000400
000	N = NFREE	00000500
000	IHB1 = IHB	00000600
000	NF = LAST + 1	00000700

000	C		00000800
000		14 DO 1 K = 2,N	00000900
000	C		00001000
000		IF(K.GT.IHB1) GO TO 2	00001100
000		M=0	00001200
000		MM=K-1	00001300
000		M1=MM*K/2	00001400
000		GO TO 3	00001500
000		2 M=K-IHB	00001600
000		MM=IHB1	00001700
000		M1=M*IHB1+LT	00001800
000		3 SUM=0.0	00001900
000		DO 4 L=1,MM	00002000
000		LL=L+M	00002100
000		JJ=LL+M1	00002200
000		LL=LL+NF-1	00002300
000		4 SUM=SUM+XK(JJ)*XK(LL)	00002400
000		1 XK(LL+1)=XK(LL+1)-SUM	00002500
000		J = NF+N-1	00002600
000	C	THIS PORTION OF SUBROUTINE PERFORMS BACK-SUBSTITUTION	00002700
000		NF=NF+N-1	00002800
000		XK(NF)=XK(NF)/XK(LAST)	00002900
000		DO 5 K=2,N	00003000
000		L=N-K+1	00003100
000		IF(L.GT.IHB1) GO TO 6	00003200
000		I=L+(L-1)*L/2	00003300
000		GO TO 7	00003400
000		6 I=L+(L-IHB)*IHB1+LT	00003500
000		7 IR=N-IHB	00003600
000		IF(L.GT.1R) GO TO 8	00003700
000		J=IHB1	00003800
000		GO TO 9	00003900
000		8 J=K-1	00004000
000		9 SUM=0.0	00004100
000		DO 10 M=1,J	00004200
000		MM=L+M	00004300
000		IF(MM.GT.IHB1) GO TO 11	00004400
000		NN=L+(MM-1)*MM/2	00004500
000		GO TO 12	00004600
000		11 NN=L+(MM-IHB)*IHB1+LT	00004700
000		12 MM=NF-N+MM	00004800
000		10 SUM=SUM+XK(NN)*XK(MM)	00004900
000		MM=NF-N+L	00005000
000		5 XK(MM)=XK(MM)/XK(1)-SUM	00005100
000		RETURN	00005200
000		END	00005300
000		WELT,SIH NASA*IPF%.SETUP,,,226231130610	
000		SUBROUTINE SETUP	00000100
000		COMMON /BLK1/ W(4),H(4),AR(4),RR(4),CR(4),AZ(4),BZ(4),CZ(4),	00000200
000		* BN(4),CN(4),DN(4),TYPEA(4,4),TYPEB(4,4),TYPEC(4,4),TYPEU(4,4),	00000300
000		* AO,BO,CO,IC,JC,KC,LC,NEL	00000400
000		W(1) = 0.3478548	00000500
000		W(2) = 0.6521452	00000600
000		W(3) = W(2)	00000700
000		W(4) = W(1)	00000800
000		H(1) = 0.8611363	00000900
000		H(2) = 0.3399810	00001000

000	H(3) = -H(2)	00001100
000	H(4) = -H(1)	00001200
000	HN(1) = -1.	00001300
000	HN(2) = 1.	00001400
000	HN(3) = 1.	00001500
000	HN(4) = -1.	00001600
000	CN(1) = -1.	00001700
000	CN(2) = -1.	00001800
000	CN(3) = 1.	00001900
000	CN(4) = 1.	00002000
000	DN(1) = 1.	00002100
000	DN(2) = -1.	00002200
000	DN(3) = 1.	00002300
000	DN(4) = -1.	00002400
000	TWOPI = 1.	00002500
000	RETURN	00002600
000	END	00002700
000	WELT,SIH NASA*IPF\$,GAUSS,,226232130610	
000	SUBROUTINE GAUSS (K,AA)	00000100
000	COMMON /BLK1/ W(4),H(4),AR(4),HR(4),CR(4),AZ(4),BZ(4),CZ(4),	00000200
000	* HN(4),CN(4),DN(4),TYPEA(4,4),TYPEB(4,4),TYPEC(4,4),TYPEU(4,4),	00000300
000	* AO,B0,C0,IC,JC,KC,LC,NFL	00000400
000	IPT = 4	00000500
000	AA = 0.	00000600
000	DO 100 I = 1,IPT	00000700
000	X = H(I)	00000800
000	DO 100 J = 1,IPT	00000900
000	Y = H(J)	00001000
000	AA = AA + W(I) * W(J) * F(K,X,Y)	00001100
000	100 CONTINUE	00001200
000	RETURN	00001300
000	END	00001400
000	FUNCTION F(K,X,Y)	00000100
000	COMMON /BLK1/ W(4),H(4),AR(4),HR(4),CR(4),AZ(4),BZ(4),CZ(4),	00000200
000	* HN(4),CN(4),DN(4),TYPEA(4,4),TYPEB(4,4),TYPEC(4,4),TYPEU(4,4),	00000300
000	* AO,B0,C0,IC,JC,KC,LC,NEL	00000400
000	COMMON /BLK9/ M,N	00000500
000	FC = AO + B0*X + C0*Y	00000600
000		00000700
000	GO TO (10,20,30,40,50,60), K	00000800
000	10 CONTINUE	00000900
000	AMAN = AR(M)*AR(N)	00001000
000	BMAN = BR(M)*AR(N) + BR(N)*AR(M)	00001100
000	AMCN = AR(M)*CR(N) + AR(N)*CR(M)	00001200
000	BMEN = BR(M)*BR(N)	00001300
000	BMCN = BR(M)*CR(N) + CR(M)*BR(N)	00001400
000	CMCN = CR(M)*CR(N)	00001500
000	GO TO 100	00001600
000	20 CONTINUE	00001700
000	AMAN = AR(M)*AZ(N)	00001800
000	BMAN = BZ(N)*AR(M) + BR(M)*AZ(N)	00001900
000	AMCN = AR(M)*CZ(N) + CR(M)*AZ(N)	00002000
000	BMEN = BR(M)*BZ(N)	00002100
000	BMCN = BR(M)*CZ(N) + CR(M)*BZ(N)	00002200
000	CMCN = CZ(N)*CR(M)	00002300
000	GO TO 100	00002400
000	30 CONTINUE	00002500

000	AMAN = AZ(M)*AZ(N)	00002600
000	AMAN = BZ(M)*AZ(N) + BZ(N)*AZ(M)	00002700
000	AMCN = AZ(M)*CZ(N) + AZ(N)*CZ(M)	00002800
000	AMBN = BZ(M)*BZ(N)	00002900
000	BMCN = BZ(M)*CZ(N) + CZ(M)*BZ(N)	00003000
000	CMCN = CZ(M)*CZ(N)	00003100
000	GO TO 100	00003200
000	40 CONTINUE	00003300
000	XY = X * Y	00003400
000	FA = 1. + BN(M)*X + CN(M)*Y + DN(M)*XY	00003500
000	FB = 1. + BN(N)*X + CN(N)*Y + DN(N)*XY	00003600
000	F = FA*FB*FC/128.	00003700
000	RETURN	00003800
000	50 F = (AR(M) + BK(M)*X + CK(M)*Y) / FC	00003900
000	RETURN	00004000
000	60 F = (AZ(M) + BZ(M)*X + CZ(M)*Y) / FC	00004100
000	RETURN	00004200
000	100 FA = AMAN + BMAN*X + AMCN*Y + BMBN*X*X + BMCN*X*Y + CMCN*Y*Y	00004300
000	F = FA/FC*.125	00004400
000	RETURN	00004500
000	END	00004600
000	WELT,SIH NASA*TPF\$.DIAGNL,,226237130610	
000	SUBROUTINE DIAGNL(XK,APF,NFREE,IHR,IHRI,LT,LAST)	00000100
000	DIMENSION XK(1),APF(1)	00000200
000	C	00000300
000	DO 200 I = 1, LAST	00000400
000	200 XK(I) = 0.	00000500
000	C	00000600
000	DO 100 J = 1, NFRE	00000700
000	IF(J.GT.IHRI) GO TO 108	00000800
000	L = (J+1) * J / 2	00000900
000	GO TO 109	00001000
000	108 L = LT + IHR * (J - IHRI)	00001100
000	109 XK(L) = 1.	00001200
000	100 CONTINUE	00001300
000	C	00001400
000	DO 110 I = J, NFRE	00001500
000	110 XK(LAST+I) = APF(1)	00001600
000	C	00001700
000	RETURN	00001800
000	END	00001900
000	WELT,SIH NASA*TPF\$.AKFA,,163614132410	
000	SUBROUTINE AKFAA(NEL,AREA)	00000100
000	PARAMETER NFI=150,NFLS=150,MX= 5000,NF=NFI*2	00000200
000	COMMON /BLK2/ ID(NF,2),IJKL(NELS,4),R(NFI),Z(NFI),KK(NELS,8)	00000300
000	COMMON /BLK7/ UDB(NF),UDDB(NF),FOV(NF),IUD(NF),DB(NF),DISP(NF),	00000400
000	* DUQT(NF)	00000500
000	DIMENSION RR(4),ZZ(4)	00000600
000	DO 100 I = 1,4	00000700
000	IN = IJKL(NFI,I)	00000800
000	JJ = IN * 2	00000900
000	II = JJ - 1	00001000
000	PR(I) = R(IN) + DISP(JJ)	00001100
000	100 ZZ(I) = Z(IN) + DISP(II)	00001200
000	AI = (RR(2)-RR(1))*(ZZ(4)-ZZ(1))-(RR(4)-RR(1))*(ZZ(2)-ZZ(1))	00001300
000	AJ = (RR(3)-RR(2))*(ZZ(4)-ZZ(2))-(RR(4)-RR(2))*(ZZ(3)-ZZ(2))	00001400
000	IF(AI.LT.0) AI = -AI	00001500

```

000      IF(AJ.LT.0) AJ = -AJ
000      AREA = (AI + AJ) / 2.
000      RETURN
000      END
000      WELT,SIH NASA*TPF$.ZERO,,,226242130610
000      SUBROUTINE ZERO(A,N,M)
000      DIMENSION A(1)
000      NM = N * M
000      DO 100 I = 1,NM
000      100 A(I) = 0.
000      RETURN
000      END
000      WXQT
000      MOVING WHEEL ON SOIL
000      65      48      5      65
000      1000      1      5
000      2000.      0.45      0.05787      29.25      2000.      0.45
000      200.      2000.      0.0006
000      36.      0.75      0.0001      0.05
000      1      24.0000      .0000      0      1
000      2      19.5000      .0000      0      1
000      3      14.0000      .0000      0      1
000      4      7.5000      .0000      0      1
000      5      .0000      .0000      1      1
000      6      24.0000      5.5000      0      0
000      7      19.5000      5.5000      0      0
000      8      14.0000      5.5000      0      0
000      9      7.5000      5.5000      0      0
000      10      .0000      5.5000      1      1
000      11      24.0000      10.5000      0      0
000      12      19.5000      10.5000      0      0
000      13      14.0000      10.5000      0      0
000      14      7.5000      10.5000      0      0
000      15      .0000      10.5000      1      1
000      16      24.0000      14.5000      0      0
000      17      19.5000      14.5000      0      0
000      18      14.0000      14.5000      0      0
000      19      7.5000      14.5000      0      0
000      20      .0000      14.5000      1      1
000      21      24.0000      18.5000      0      0
000      22      19.5000      18.5000      0      0
000      23      14.0000      18.5000      0      0
000      24      7.5000      18.5000      0      0
000      25      .0000      18.5000      1      1
000      26      24.4000      22.5000      0      0
000      27      19.5000      22.5000      0      0
000      28      14.0000      22.5000      0      0
000      29      7.5000      22.5000      0      0
000      30      .0000      22.5000      1      1
000      31      25.4500      26.5000      0      0
000      32      20.0000      26.5000      0      0
000      33      14.0000      26.5000      0      0
000      34      7.5000      26.5000      0      0
000      35      .0000      26.5000      1      1
000      36      26.8000      30.0000      0      0
000      37      21.0000      30.0000      0      0
000      38      14.0000      30.0000      0      0

```

000	39	7.5000	30.0000	0	0
000	40	.0000	30.0000	1	1
000	41	29.2500	33.5000	0	0
000	42	22.5000	33.5000	0	0
000	43	15.2500	33.5000	0	0
000	44	8.2500	33.5000	0	0
000	45	.0000	33.5000	1	1
000	46	29.2500	37.5000	0	0
000	47	23.5000	37.5000	0	0
000	48	16.5000	37.5000	0	0
000	49	9.0000	37.5000	0	0
000	50	.0000	37.5000	1	1
000	51	29.2500	42.0000	0	0
000	52	23.5000	42.0000	0	0
000	53	16.5000	42.0000	0	0
000	54	9.0000	42.0000	0	0
000	55	.0000	42.0000	1	1
000	56	29.2500	47.5000	0	0
000	57	23.5000	47.5000	0	0
000	58	16.5000	47.5000	0	0
000	59	9.0000	47.5000	0	0
000	60	.0000	47.5000	1	1
000	61	29.2500	55.0000	0	1
000	62	23.5000	55.0000	0	1
000	63	16.5000	55.0000	0	1
000	64	9.0000	55.0000	0	1
000	65	.0000	55.0000	1	1
000	1	2	7	6	1
000	2	7	12	11	6
000	3	12	17	16	11
000	4	17	22	21	16
000	5	22	27	26	21
000	6	27	32	31	26
000	7	32	37	36	31
000	8	37	42	41	36
000	9	42	47	46	41
000	10	47	52	51	46
000	11	52	57	56	51
000	12	57	62	61	56
000	13	3	8	7	2
000	14	8	13	12	7
000	15	13	18	17	12
000	16	18	23	22	17
000	17	23	28	27	22
000	18	28	33	32	27
000	19	33	38	37	32
000	20	38	43	42	37
000	21	43	48	47	42
000	22	48	53	52	47
000	23	53	58	57	52
000	24	58	63	62	57
000	25	4	9	8	3
000	26	9	14	13	8
000	27	14	19	18	13
000	28	19	24	23	18
000	29	24	29	28	23
000	30	29	34	33	28

000	31	34	39	58	55
000	32	39	44	43	58
000	33	44	49	48	45
000	34	49	54	53	48
000	35	54	59	58	55
000	36	59	64	63	58
000	37	5	10	9	4
000	38	10	15	14	9
000	39	15	20	19	14
000	40	20	25	24	19
000	41	25	30	29	24
000	42	30	35	34	29
000	43	35	40	39	34
000	44	40	45	44	39
000	45	45	50	49	44
000	46	50	55	54	49
000	47	55	60	59	54
000	48	60	65	64	59
000	21	-13.4152		-9.5128	
000	26	-55.2063		-26.6758	
000	31	-98.2728		-27.7647	
000	36	-89.0538		-8.3968	
000	41	-32.0671		2.8452	

LEEJK ACCOUNT: UAH31736502F PROJECT: NASA

APPENDIX 4

DATA INPUT FORMAT

Card 1: FORMAT (20 A4)

TITLE - Title of the problem

Card 2: FORMAT (10I5)

- (1) INODE - No. of nodes
- (2) NELEM - No. of elements
- (3) NAPC - No. of applied point load
- (4) NBC - No. of free nodes

Card 3: FORMAT (10I5)

- (1) ISW(1) = 0, static analysis
= N, dynamic analysis for N steps
= -N, static elasto-plastic analysis for
N load increments
- (2) ISW(2) = 0, Elastic analysis
= -1, Viscoelastic analysis
= 1, Viscoelastoplastic analysis
- (3) ISW(3) = M, Print for each Mth time step
- (4) ISW(4) = 0
- (5) ISW(5) = No. of rigid elements

Card 4: FORMAT (6F13.6)

- (1) E - Modulus of elasticity for rigid element
- (2) XNU - Poisson's ratio for rigid element
- (3) DENS - Soil density
- (4) DEPTH - Maximum soil depth

(5) ES - Modulus of elasticity for soil

(6) XNUS - Poisson's ratio for soil

Card 5: FORMAT (6F13.6)

(1) AT = $T_{(r)} \times E_s$, where $T_{(r)}$ is the soil relaxation time in seconds

(2) XK - Modulus of elasticity for soil

(3) DELT - Magnitude of time step in seconds

Card 6: FORMAT (6F13.6)

(1) PHI - Angle of internal friction

(2) VOIDI - Initial void ratio

(3) CAPA - Swelling index

(4) RAMD - Compression index

Cards 7: FORMAT (5x, 2F10.4, 2I5)

(1) Z(I) - * Z - coordinate value of Node I

(2) R(I) - R - coordinate value of Node I

(3) IZ = 0 if free to z-direction

≠ 1 if not

(4) IR = 0 if free to R-direction

≠ 1 if not

Repeat INODE times in the order of node number

* Upward Z is positive

Cards 8: FORMAT (5x, 5I5)

4 corner nodes of an element in counter clockwise. Repeat NELEM times in the order of element number. Note that rigid element should be numbered first.

Card(s) 9: FORMAT (I5, 2F10.4)

(1) NODE - Node number with applied load

(2) PZ - Z-component

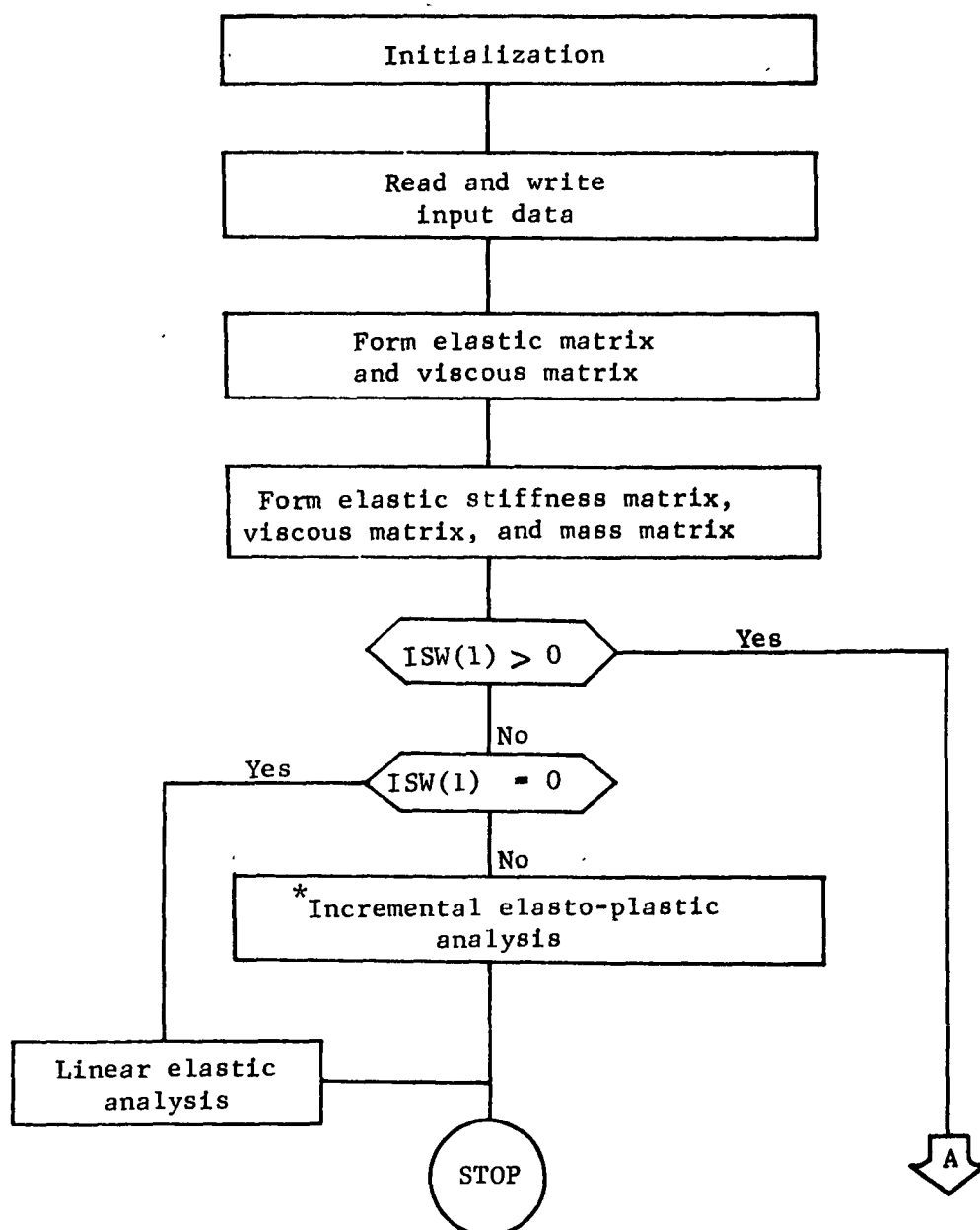
(3) PR - R-component

Note that the regular sign convention of theory of elasticity
is used for input quantity.

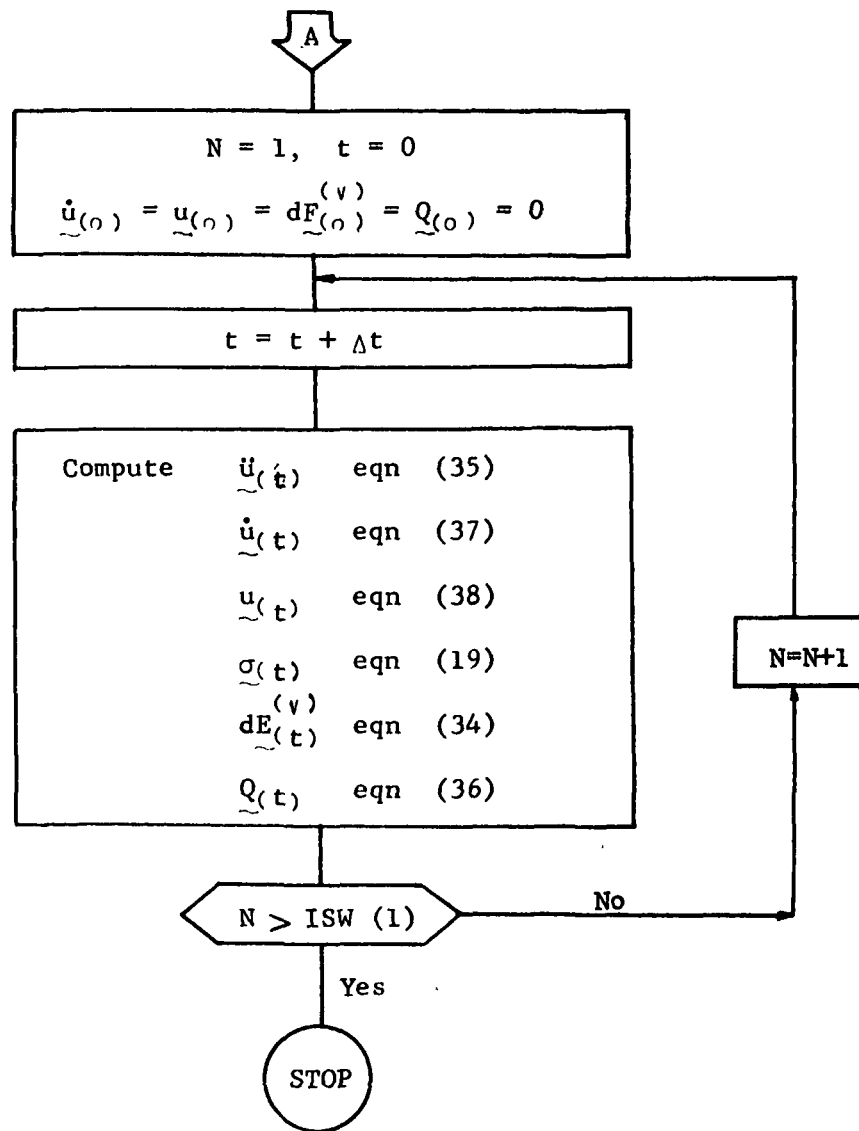
Repeat NAPC times.

APPENDIX 5

FLOW CHART

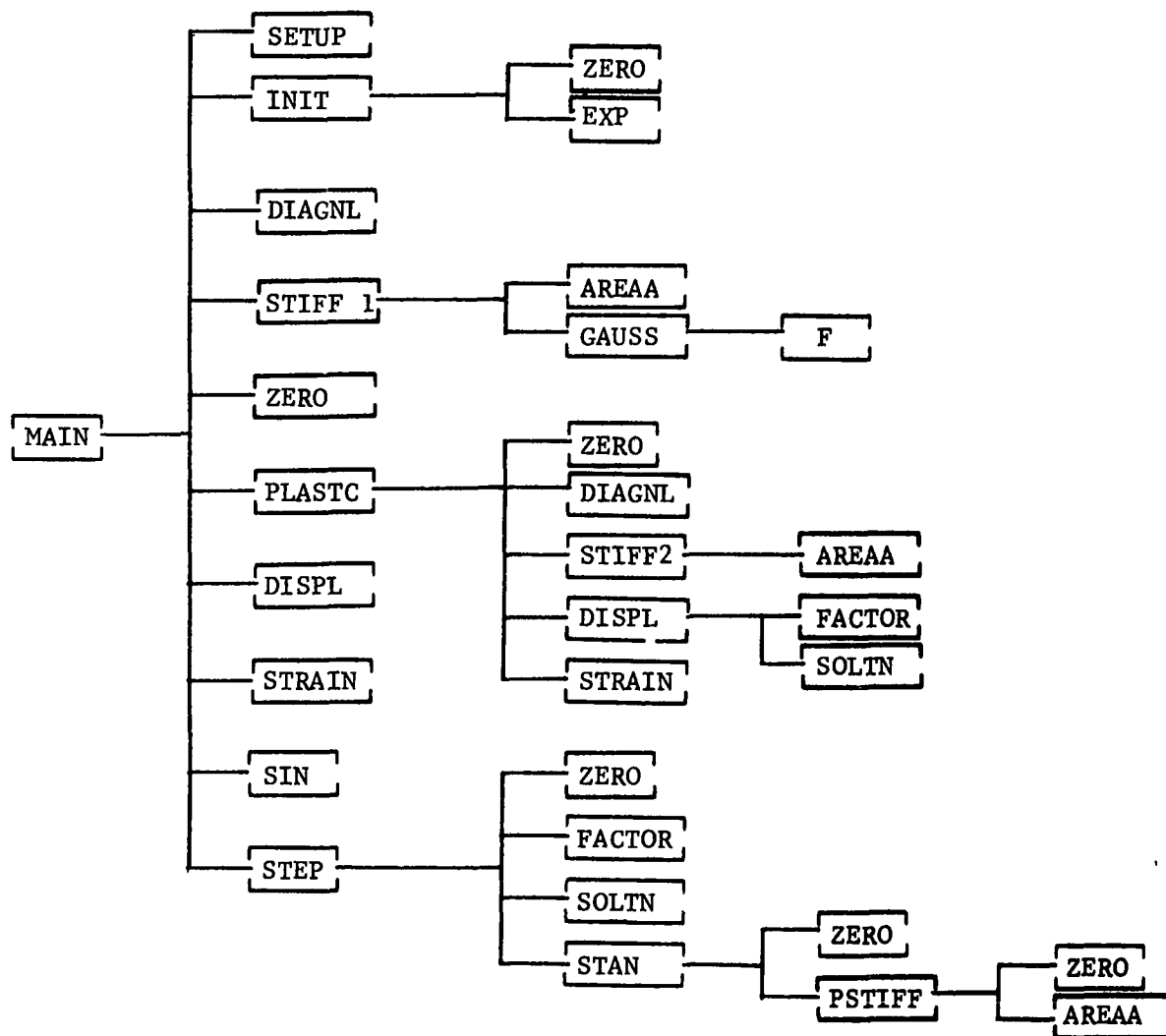


*For incremental elasto-plastic analysis, see App. 3 of Part I.



APPENDIX 6

SUBROUTINE ORGANIZATION CHART



APPENDIX 7

DESCRIPTIONS OF SUBROUTINES

Subroutine Name	Descriptions
AREAA	Computes the cross sectional area of an element
DIAGNL	Clears one dimensional array and puts 1's on diagonal.
DISPL	Calls FACTOR and SOLTN, and prints displacement vector.
F	Functions to be integrated
FACTOR	Factors (forward substitution) a given simultaneous equations
GAUSS	Integrates by the Gaussian quadrature
INIT	Forms elastic matrix and viscous matrix
PSTIFF	Checks for yielding and forms plastic stiffness matrices for yielded elements
SETUP	Initializes integration constants
SOLTN	Backward substitution is performed to give a set of solutions to the given simultaneous equations.
STEP	Integrates the equation of motion by step integration scheme for dynamic analysis
STIFF 1	Forms elastic stiffness matrix, consistent mass matrix, and viscous matrix. Also assembles in global form applying the boundary conditions.
STIFF 2	Checks for yielding and forms elasto-plastic stiffness matrix, and assembles in global form applying the boundary conditions.
STRAIN	Computes strains, stresses, and principal stresses (compression is positive).
ZERO	Clears any given matrix